Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group**.

Names:

Boolean Algebra

Logic, Sets, and Boolean Algebra

Logic ($p \wedge q$), sets ($P \cap Q$), and now boolean algebra ($p \cdot q$) have a lot in common with each other. In fact, sometimes it can be useful to convert a problem from one type to another in order to learn more about a problem. Here are the "translations":

	Logic	**Sets**	**Boolean Algebra**
Variables	p, q, r	A, B, C	a, b, c
"and" operation	$\wedge$	$\cap$	$\cdot$
"or" operation	$\vee$	$\cup$	$+$
"not" operation	$\neg$	$'$	$'$
"-" operation	$a \wedge \neg b$	$A - B$	$a \cdot b'$
Special	Tautology	Universal set U	1
	Contradiction	Empty set $\emptyset$	0

Example:
Rephrase the following Logic operation using Set and Boolean Algebra notations: $(p \land q) \lor r$

- Logic: $(p \land q) \lor r$
- Sets: $(P \cap Q) \cup R$
- Boolean algebra: $(p \cdot q) + r$

Question 1

Rewrite the following using Boolean Algebra notation:

a. $p \land q$

b. $p \lor q$

c. $\neg p$

d. $(p \land \neg q) \lor p$

e. $\neg(\neg p)$

f. $(p \land \neg q) \lor p \equiv p$

g. $(A - B)$

h. $A' \cup (A \cap B)$

i. $(A - B)' = A' \cup (A \cap B)$

Boolean Algebra properties

Boolean Algebra Properties [a]

Commutative	$a \cdot b = b \cdot a$	$a + b = b + a$
Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(a + b) + c = a + (b + c)$
Distributive	$a \cdot (b + c)$ $= (a \cdot b) + (a \cdot c)$	$a + (b \cdot c)$ $= (a + b) \cdot (a + c)$
Identity	$a \cdot 1 = a$	$a + 0 = a$
Negation	$a + a' = 1$	$a \cdot a' = 0$
Double negative	$(a')' = a$	
Idempotent	$a \cdot a = a$	$a + a = a$
DeMorgan's laws	$(a \cdot b)' = a' + b'$	$(a + b)' = a' \cdot b'$
Universal bound	$a + 1 = 1$	$a \cdot 0 = 0$
Absorption	$a \cdot (a + b) = a$	$a + (a \cdot b) = a$
Complements of 1 and 0	$1' = 0$	$0' = 1$

[a] From Discrete Mathematics, Ensley and Crawley

Examples [a]

Example 1: Simplify $(a+1)\cdot(a+0)$

$(a+1)\cdot(a+0)$	
$=1\cdot(a+0)$	universal bound
$=(a+0)\cdot 1$	commutative
$=a+0$	identity
$=a$	identity

Example 2: Simplify $a\cdot(a'+b)$

$a\cdot(a'+b)$	
$=a\cdot a'+a\cdot b$	distributive
$=0+a\cdot b$	negation
$=a\cdot b+0$	commutative
$=a\cdot b$	identity

Example 3: Show that $a\cdot b+b\cdot c=(a+c)\cdot b$

$a\cdot b+b\cdot c$	
$=b\cdot a+b\cdot c$	commutative
$=b\cdot(a+c)$	distributive
$=(a+c)\cdot b$	commutative

Example 4: Show that if $a'+b=1$ then $a\cdot b'=0$.

$a'+b$	
$=(a')'\cdot b'$	double negative
$=(a'+b)'$	DeMorgan
$=(1)'$	Since $a'+b=1$
$=0$	Complements

[a] From Discrete Mathematics, Ensley & Crawley, 3.4

Question 2

Simplify the following equations using the Boolean Algebra properties.

a. $ab + ab'$

b. $cd \cdot c'd'$

c. $e \cdot (e + f)$

d. $g \cdot (h + i)$

e. $j + (k \cdot l)$

f. $m + (no + no')$

Example

Using the Boolean Algebra properties, transform the left-hand side of each equation to the right-hand side.

Show that $ba + ba' = b$

	Property		Step
1.	Distributive	$a \cdot (b + c) = (ab) + (ac)$	$ba + ba' \to b \cdot (a + a')$
2.	Negation	$a + a' = 1$	$b \cdot (a + a') \to b \cdot (1)$
3.	Identity	$a \cdot 1 = a$	$b \cdot (1) \to b \cdot 1 = b$

Therefore $ba + ba' = b$

Question 3

Using the Boolean Algebra properties, transform the left-hand side of each equation to the right-hand side.

c. $x'yz + x'y'z + xyz' + xy'z' = xz' + x'z$
(Group up two terms at a time: $xyz' + xy'z'$ and $x'yz + x'y'z$.)

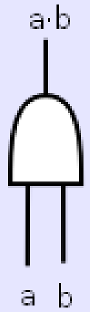
d. $xyz + xyz' + x'yz + x'yz' = y$

By Rachel Singh, last updated November 1, 2018

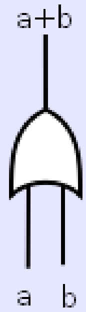
Logic Circuits

We are going to be using logic gates as one way to represent our Boolean Algebra expressions graphically. The gates that we will be using are:

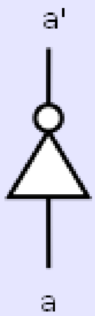
AND $\cdot$



OR $+$



NOT $'$



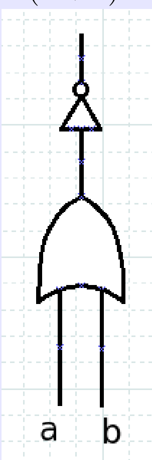
a	b	$a \cdot b$
0	0	0
0	1	0
1	0	0
1	1	1

a	b	$a + b$
0	0	0
0	1	1
1	0	1
1	1	1

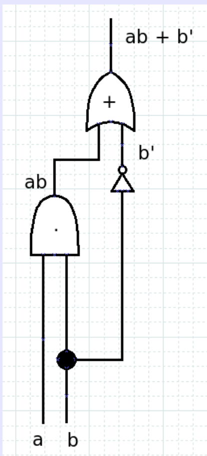
a	a'
0	1
1	0

Additionally, we can connect gates together in order to build an expression. For example:

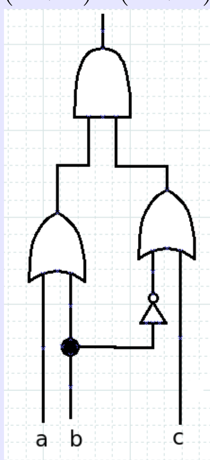
$(a + b)'$



$ab + b'$

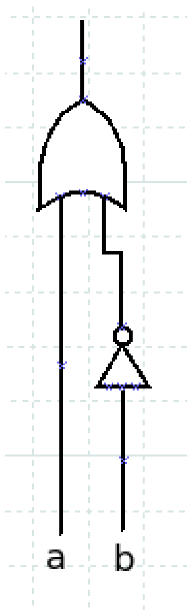


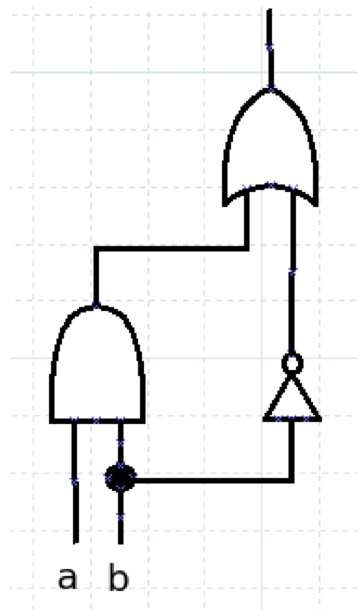
$(a + b) \cdot (b' + c)$

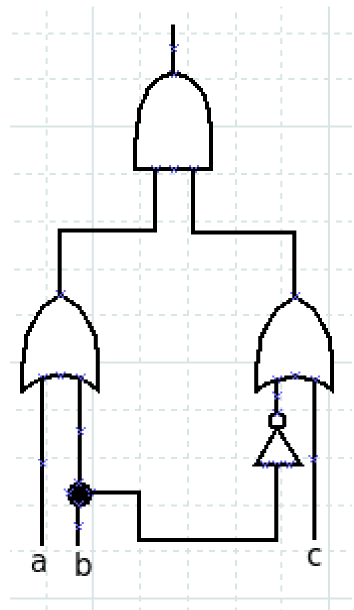


Question 4

Write out the Boolean expression that describes each diagram:







Question 5

Draw a circuit diagram for the following Boolean expressions:

a. $a + b'$	b. $a' \cdot b'$	c. $a + (b \cdot c)$