Boolean Algebra

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About

Logic, Sets, and Boolean Algebra are related, and we can translate problems between each of these…

Topics

1. Boolean Algebra
2. Properties
3. Logic Circuits

Boolean Algebra

1. Boolean Algebra

Logic ($p \wedge q$), sets ($P \cap Q$), and now boolean algebra ($p \cdot q$) have a lot in common with each other. In fact, sometimes it can be useful to convert a problem from one type to another in order to learn more about a problem.

Notes

1. Boolean Algebra

"Translations":

	Logic	Sets	Boolean Algera
Variables	p, q, r	A, B, C	a, b, c
"and"	∧	∩	·
"or"	∨	∪	+
"not" / negation	¬	‘	‘
"difference" / -	p ∧ ¬q	A – B	a · b’

Notes

1. Boolean Algebra

Example: Translate $(p \wedge q) \vee r$ into Set notation and Boolean Algebra notation.

Notes

1. Boolean Algebra

Example: Translate $(p \wedge q) \vee r$ into Set notation and Boolean Algebra notation.

Set notation: $(P \cap Q) \cup R$

Boolean algebra: $(p \cdot q) + r$

or $pq + r$

Notes

1. Boolean Algebra

Example: Translate A – B to Logic and Boolean Algebra notation.

Notes

1. Boolean Algebra

Example: Translate A – B to Logic and Boolean Algebra notation.

Logic: a ∧ ¬b

Boolean algebra: a·b’

Notes

Properties

2. Properties

Just like with Sets and Logic, there are a set of properties that hold, that allow us to manipulate our Boolean algebra statements to see what equations are equivalent.

Notes

2. Properties

Commutative

$$a \cdot b = b \cdot a$$

$$a + b = b + a$$

Notes

Commutative
$a \cdot b = b \cdot a$
$a + b = b + a$

2. Properties

Associative

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

$$(a + b) + c = a + (b + c)$$

Notes

Commutative

$a \cdot b = b \cdot a$

$a + b = b + a$

Associative

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$(a + b) + c = a + (b + c)$

Distributive

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

Notes

Commutative

$a \cdot b = b \cdot a$

$a + b = b + a$

Associative

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$(a + b) + c = a + (b + c)$

Distributive

$a \cdot (b + c)$

$= (a \cdot b) + (a \cdot c)$

$a + (b \cdot c)$

$= (a + b) \cdot (a + c)$

2. Properties

Identity

$a \cdot 1 = a$

$a + 0 = a$

Notes

Identity
$a \cdot 1 = a$
$a + 0 = a$

Negation

$$a + a' = 1$$

$$a \cdot a' = 0$$

Notes

Identity
$a \cdot 1 = a$
$a + 0 = a$

Negation
$a + a' = 1$
$a \cdot a' = 0$

2. Properties

Double Negative

$$(a')' = a$$

Notes

Identity

$a \cdot 1 = a$

$a + 0 = a$

Negation

$a + a' = 1$

$a \cdot a' = 0$

2. Properties

Idempotent

$$a \cdot a = a$$

$$a + a = a$$

Notes

Identity
$a \cdot 1 = a$
$a + 0 = a$

Negation
$a + a' = 1$
$a \cdot a' = 0$

Idempotent
$a \cdot a = a$
$a + a = a$

DeMorgan's Laws

$$(a \cdot b)' = a' + b'$$

$$(a + b)' = a' \cdot b'$$

Notes

DeMorgan's Laws
$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

2. Properties

Universal bound

$$a + 1 = 1$$

$$a \cdot 0 = 0$$

Notes

DeMorgan's Laws
$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

Universal bound
$a + 1 = 1$
$a \cdot 0 = 0$

2. Properties

Absorption

$$a \cdot (a + b) = a$$

$$a + (a \cdot b) = a$$

Notes

DeMorgan's Laws

$(a \cdot b)' = a' + b'$

$(a + b)' = a' \cdot b'$

Universal bound

$a + 1 = 1$

$a \cdot 0 = 0$

Absorption

$a \cdot (a + b) = a$

$a + (a \cdot b) = a$

Complements of 1 and 0

$$1' = 0$$

$$0' = 1$$

Notes

DeMorgan's Laws

$(a \cdot b)' = a' + b'$

$(a + b)' = a' \cdot b'$

Universal bound

$a + 1 = 1$

$a \cdot 0 = 0$

Absorption

$a \cdot (a + b) = a$

$a + (a \cdot b) = a$

2. Properties

Example: Simplify the following expression.

$$(a+1)\cdot(a+0)$$

Notes

$a \cdot (b + c)$
$= (a \cdot b) + (a \cdot c)$

$a + (b \cdot c)$
$= (a + b) \cdot (a + c)$

$a + a' = 1$
$a \cdot a' = 0$

$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

$a \cdot (a + b) = a$
$a + (a \cdot b) = a$

2. Properties

Example: Simplify the following expression.

	$(a+1)\cdot(a+0)$	
=	$1 \cdot (a+0)$	**Universal Bound**
=	$(a+0) \cdot 1$	**Commutative**
=	$a + 0$	**Identity**
=	a	**Identity**

Notes

$a \cdot (b + c)$
$= (a \cdot b) + (a \cdot c)$

$a + (b \cdot c)$
$= (a + b) \cdot (a + c)$

$a + a' = 1$
$a \cdot a' = 0$

$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

$a \cdot (a + b) = a$
$a + (a \cdot b) = a$

2. Properties

Example: Simplify the following expression.

$$a \cdot (a' + b)$$

Notes

$a \cdot (b + c)$
$= (a \cdot b) + (a \cdot c)$

$a + (b \cdot c)$
$= (a + b) \cdot (a + c)$

$a + a' = 1$
$a \cdot a' = 0$

$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

$a \cdot (a + b) = a$
$a + (a \cdot b) = a$

2. Properties

Example: Simplify the following expression.

	$a \cdot (a' + b)$	
=	$a \cdot a' + a \cdot b$	**Distributive**
=	$0 + a \cdot b$	**Negation**
=	$a \cdot b + 0$	**Commutative**
=	$a \cdot b$	**Identity**

Notes

$a \cdot (b + c)$
$= (a \cdot b) + (a \cdot c)$

$a + (b \cdot c)$
$= (a + b) \cdot (a + c)$

$a + a' = 1$
$a \cdot a' = 0$

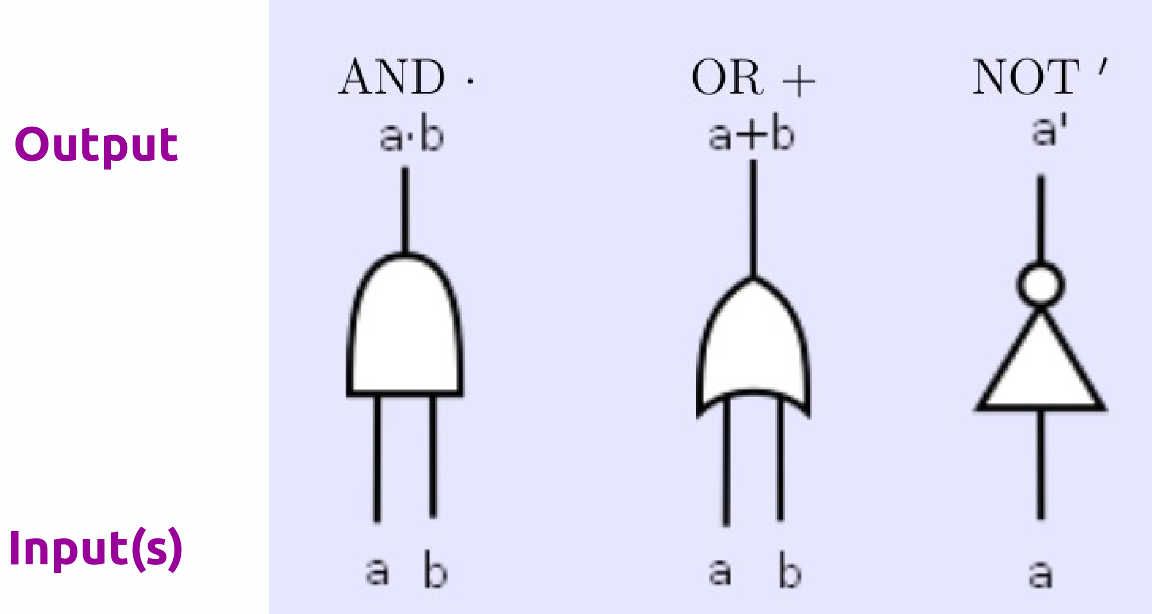
$(a \cdot b)' = a' + b'$
$(a + b)' = a' \cdot b'$

$a \cdot (a + b) = a$
$a + (a \cdot b) = a$

Logic Gates

3. Logic Gates

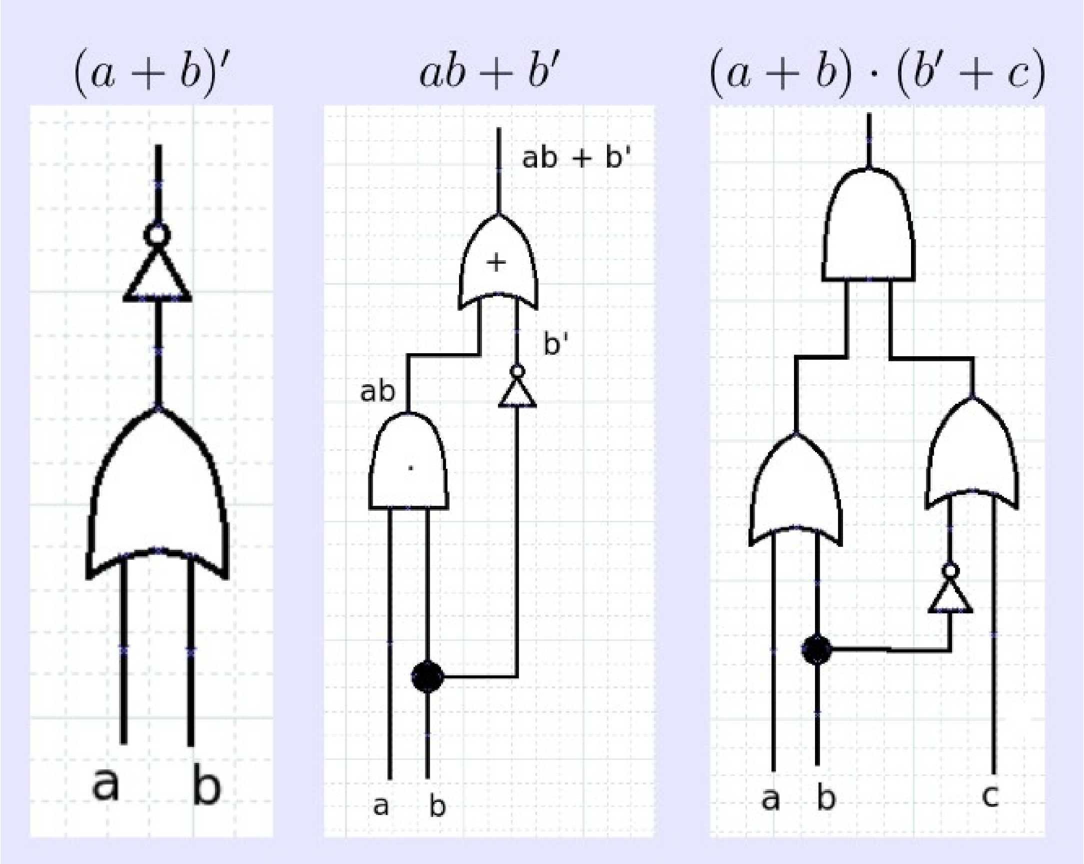
We are going to be using logic gates as one way to represent our Boolean Algebra expressions graphically. The gates that we will be using are:



Notes

3. Logic Gates

We can use logic gates to graphically represent a boolean algebra equation.



Notes

Conclusion