

Name: _____

Chapter 5 Exam

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Grading Scheme

#	Question	Weight	Points Received
1	Building a Committee	12%	
2	Dealing Cards	12%	
3	Binary Sequence	12%	
4	Elections	20%	
5	License Plate	12%	
6	Buying Fruit	12%	
7	Defective Toys	20%	
8	Extra Credit Question	+2%	
9	Extra Credit Question	+2%	

Cheatsheet

Terminology:

Structure formulas:

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list*	yes	no	$C(r + n - 1, r)$
Permutations of length r	no	yes	$P(n, r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n, r) = \frac{n!}{r!(n-r)!}$

* If you set $r = 1$'s and $n - r = 0$'s, then you can compute the result with $C(n, r)$.

The Rule of Sums:

In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, **it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.** ^a

The Rule of Sums with Overlap:

If the list to count can be split into two pieces of size x and y , and the pieces have z objects in common, then the original list has $x + y - z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B . ^b

The Rule of Products:

In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, **it is the idea that if there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions.** ^c

^aFrom https://en.wikipedia.org/wiki/Rule_of_sum

^bFrom Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

^cFrom https://en.wikipedia.org/wiki/Rule_of_product

The Rule of Complements:

If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$.^a

Binary sequence theorem The number of binary sequences with r 1's and $n - r$ 0's is $C(n, r)$ or $C(n, n - r)$.^b

Theorem 1: Fundamental Theorem of Sums and Differences

For any sequence $\{s_n\}$ with first differences $\Delta_k = s_{k+1} - s_k$, and any $n \geq 1$,

$$s_n - s_0 = \sum_{k=0}^{n-1} \Delta_k$$

or

$$s_n = \sum_{k=0}^{n-1} \Delta_k + s_0$$

^aFrom Discrete Mathematics, Ensley and Crawley, page 390

^bFrom Discrete Mathematics, Ensley and Crawley, page 409

Question 1: Building a Committee

0 1 2 3 4

In a class of 25 students, how many ways can a 3-person committee be formed?

Question 2: Dealing Cards

0 1 2 3 4

How many ways are there to select 4 cards from a standard deck of 52, with no replacement?

Question 3: Binary Sequence

0 1 2 3 4

How many binary sequences can be generated of length 5 that have exactly 2 ones? Solve by writing out all possibilities AND using the Binary Sequence Theorem.

Binary sequence theorem

The number of binary sequences with r 1's and $n - r$ 0's is $C(n, r)$ or $C(n, n - r)$.

Question 5: License Plate

0 1 2 3 4

For some state, the license plate requires 4 letters (A-Z is 26 letters), followed by 2 numbers (0-9 is 10 numbers). How many different ways are there to build a license plate?

Question 6: Buying Fruit

0 1 2 3 4

You're purchasing a box of 20 pieces of fruit from a store that sells 6 types of fruit. How many ways can you buy 20 pieces of fruit?
Remember: 1's are separators and 0's are fruits to buy.

Question 7: Defective Toys

0 1 2 3 4

A box contains 15 toys. 3 toys in the box are broken. Let's say a kid will take 5 toys from the box...

- a. How many ways are there for the kid to select 5 toys with no restrictions?

- b. How many ways will there be exactly 1 broken toy?

- c. How many ways will contain all the broken toys?

- d. How many ways will contain no broken toys?

Extra credit

Question 8: Finding the Closed Formula 0 1 2 3 4

Build a difference table and find the formulas for the sequence:

2, 5, 8, 11, 14, ...

Question 9: Adjacency Matrix 0 1 2 3 4

Draw the directed graph for the given adjacency matrix.

		Columns									
		1	2	3	4	5					
Rows	1	0	1	0	0	1					
	2	1	0	0	1	1	1				
	3	1	0	1	1	0					
	4	0	0	0	0	1					
	5	0	0	0	0	0					

4		
•		
1	5	2
•	•	•
3		
•		