Name: _____

Chapter 5 EXAM B:

This exam covers concepts from Chapter 5. Questions will be labelled by the section from which they are based.

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Grading breakdown:

Question	Weight	Points Received	Weighted Score
1	4%		
2	4%		
3	4%		
4	4%		
5	15%		
6	16%		
7	16%		
8	15%		
9	15%		
10	7%		
11	+1%		
12	+1%		
13	+1%		
14	+1%		
15	+1%		

Exam

Question 0

1. Name one goal you have for this class

2. Name one discovery from this class that surprised you

3. Name one success you had so far in this class

Hints

This is the lucky math-moose.

Lucky math-moose reminds you that if you get stuck on a problem, skip it and return to it later. Also sometimes it helps to sketch out the problem so you can visualize it better.

(Scratch page)

Identification

For these questions, identify the type of structure.
Do not solve the problem.

4% Question 1: Identification □ 0 □ 1 □ 2 □ 3 □ 4

How many ways can we fill a box of donuts if the box can store 12 donuts, and there are 6 types of donuts to choose from?

The structure is:

4% Question 2: Identification □ 0 □ 1 □ 2 □ 3 □ 4

There are three red flowers, two blue flowers, and six purple flowers in the garden. In how many ways can four be selected?

The structure is:

4% Question 3: Identification □ 0 □ 1 □ 2 □ 3 □ 4

How many arrangements are there of the letters in the word CATS?

The structure is:

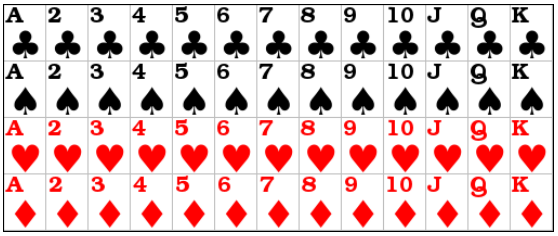
4% Question 4: Identification □ 0 □ 1 □ 2 □ 3 □ 4

In the cafeteria, there are 5 types of meals to choose from. How many ways are there to select a meal for yourself, your best friend, and your second-best-friend?

The structure is:

Combinatorics

Hint
Before solving the problems, try to identify the **structure**, the value for n, and the value for r, and whether you will need any special rules!



15% Question 5: Cards □ 0 □ 1 □ 2 □ 3 □ 4

Ethan is going to select 3 cards from a standard deck.

a. How many different results are there, with no restrictions?

_____ _____ _____

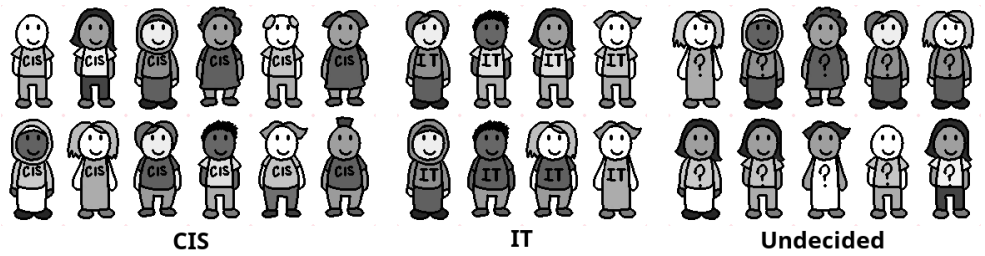
(Continued on next page...)

b. In how many of these outcomes do the first and last cards have the same value?

c. In how many ways do all 3 cards have the same suit?

16% Question 6: Elections □ 0 □ 1 □ 2 □ 3 □ 4

In a CS 211 class there are 30 students: 12 CIS majors, 8 IT majors, and 10 undeclared majors. In how many ways can we elect a president, vice president, and secretary for each of the following situations?



a. No restrictions.

b. Three officers are all the same major.

(Continued on next page...)

c. None of the officers are undeclared.

d. *At least* one officer must be undeclared.

16% Question 7: Defective computers □ 0 □ 1 □ 2 □ 3 □ 4

Suppose a shipment of 50 computers contain 6 defective computers, and we are choosing a sample of 10 computers to test out.

a. How many different samples are there?

b. Of these, how many samples contain all 6 defective computers?

(Continued on next page...)

c. How many samples contain no defective computers?

d. How many samples contain one or more defective computer?

15% Question 8: Burgies □ 0 □ 1 □ 2 □ 3 □ 4

Vance and Paul are going to the burger store for some burgers. There are 10 types of burgers at the burger store. If Paul buys a burger for himself and for his buddy Vance, how many different ways can he buy two burgers, assuming it's OK if Vance and Paul end up with the same burger?

Type 1

Type 2

Type 3

Type 4

Type 5

Type 6

Type 7

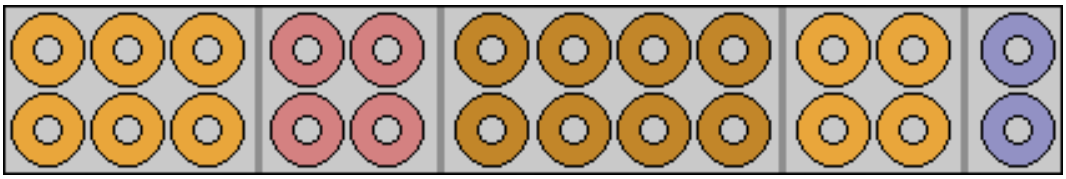
Type 8

Type 9

Type 10

15% Question 9: Box-o-donuts □ 0 □ 1 □ 2 □ 3 □ 4

Alexa is going to buy donuts for her CS 211 class. Including herself and the teacher, there are 24 people. The local donut shop has 5 types of donuts, and it luckily has a box that will store 24 donuts and 4 separators to keep the donuts from merging flavors. How many different ways are there to select 24 donuts from the 5 types?



Sequences

7% Question 10: Finding a formula □ 0 □ 1 □ 2 □ 3 □ 4

Find the closed formula for the sequence

$$3, 10, 17, 24, 31$$

.

n					
a_n					
$\triangle_n^1$					

Hints
For any sequence s_n with first differences $\triangle_k^1 = s_{k+1} - s_k$, and any $n \geq 1$...

$$s_n = s_0 + \sum_{k=0}^{n-1} \triangle_k^1$$

Extra credit

+1% Question 11: Proof (5.5) □ 0 □ 1 □ 2 □ 3 □ 4

Use the recurrence relation

$$P(n, r) = n \cdot P((n-1), (r-1))$$

(with $P(n, 0) = 1$ for all $n \geq 0$) to prove for all $n \geq 1$, $P(n, 1) = n$

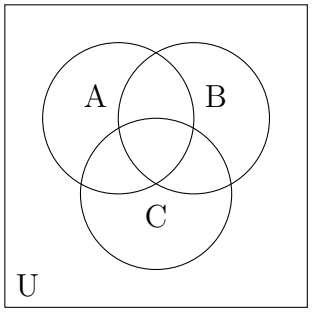
+1% Question 12: Quantified predicates (1.4) □ 0 □ 1 □ 2 □ 3 □ 4

Write the following as a quantified predicate:
"For all integers x, there exists some integer y, such that $x - y < 0$."

+1% Question 13: Venn Diagram (3.1) □ 0 □ 1 □ 2 □ 3 □ 4

Draw a Venn diagram for the following:

$$A \cup (B - C)$$



+1% Question 14: Composition Operation (4.2) □ 0 □ 1 □ 2 □ 3 □ 4

Given $f(x) = 2x + 1$ and $g(y) = y^2 - 1$, what is $(f \circ g)(z)$?

+1% Question 15: Other □ 0 □ 1 □ 2 □ 3 □ 4

Draw a cat

Cheat sheet

The Rule of Product When you see the "and" keyword, you should be multiplying the different outcomes.

The Rule of Sum When you see the "or" keyword, you should be adding the different outcomes.

The Rule of Sum with Overlap When you're choosing *this* "or" *that*, and this & that have overlap, you calculate it with $this + that - overlap$.

The Rule of Complements If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$.

Structures

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r+n-1, r)$
Permutations of length r	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$

Binary sequences

The number of binary sequences with r 1's and $n-r$ 0's is $C(n,r)$ or $C(n, n-r)$.

Fruits: When building a problem where you have t types of fruits and you are selecting p pieces of fruit, then the amount of separators, the 1's, is $t+1$, and the amount of fruits, the 0's, is p, and the total amount of spots you're filling is $n = p + t + 1$, then you can pass in $r = t + 1$ (the # of 1's) or $r = p$ (the # of 0's).