Chapter 1 EXAM PREVIEW

Exam preview, key, and cheat sheet included. There are more questions on this preview than there are on the exam. I'm just giving extra problems so that you can more thoroughly study each topic.

There will be the following: 4 identification problems (identify the structure, but don't solve), 5 combinatorics problems (word problems, solve), 1 sequence problem (find the closed formula), and 5 extra credit problems.

Questions

For these questions, identify the type of structure and whether the rule of products, the rule of sums, or the rule of sums with overlap will be used.
Then solve the problem.

Question 1

A certain mid-Atlantic state has a simple rule for its license plates: Use three letters followed by four digits. [1]

Question 2

In how many ways can a club with 17 members elect a president, vice president, and secretary (assuming no person can fill more than one office) given no restrictions? [2]

Question 3

An organization has 8 math majors, 12 computer science majors, and 6 science majors. In how many ways can the organization elect a president, a vice president, and a secretary, given that all three officers must be the same major? [3]

[1] Chapter 5.2, Exercise 3 from Discrete Mathematics, Ensley & Crawley
[2] Chapter 5.2, Exercise 6 from Discrete Mathematics, Ensley & Crawley
[3] Chapter 5.2, Exercise 8 from Discrete Mathematics, Ensley & Crawley

Question 4

How many ways can you rearrange the letters in EMPHATIC? [4]

Question 5

How many subsets of $\{1, ..., 6\}$ have an odd number of elements? [5]

Question 6

A blackjack hand consists of two cards, the first of which is dealt face-down and the second face-up. In how many blackjack hands is the face-up card an ace and the face-down card a 10? [6]

Question 7

How many committees of three people can be formed from a club with 17 members? [7]

Question 8

A bag contains a dozen oranges, two of which are rotten. A sample of three oranges is taken from the bag. How many ways can the sample be taken? [8]

[4] Chapter 5.2, Exercise 13 from Discrete Mathematics, Ensley & Crawley
[5] Chapter 5.3, Exercise 10 from Discrete Mathematics, Ensley & Crawley
[6] Chapter 5.2, Exercise 14 from Discrete Mathematics, Ensley & Crawley
[7] Chapter 5.3, Exercise 13 from Discrete Mathematics, Ensley & Crawley
[8] Chapter 5.3, Exercise 17 from Discrete Mathematics, Ensley & Crawley

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Question 9

A ternary sequence is a sequence of digits chosen from {0, 1, 2}. How many ternary sequences of length 12 can we select, without restrictions?

Question 10

How many binary sequences of length 8 are there that have exactly six 1's? [9]

Question 11

How any ways are there to fill a box with 20 pieces of fruit at a store that sells 3 types of fruit? [10]

Sequences

Hints

For any sequence s_n with first differences $\triangle_k^1 = s_{k+1} - s_k$, and any $n \geq 1$...

$$s_n = s_0 + \sum_{k=0}^{n-1} \triangle_k^1$$

Question 12

Find the closed formula for the following sequence. Make sure to build a difference table. [11]

$$2, 5, 8, 11, 14$$

[9] Chapter 5.4, Exercise 1 from Discrete Mathematics, Ensley & Crawley
[10] Chapter 5.4, Exercise 16 from Discrete Mathematics, Ensley & Crawley
[11] Chapter 5.6, Example 1 from Discrete Mathematics, Ensley & Crawley

Exam key

1. You will select the letters and the digits separately, and combine them with "and", so you will be using the product rule. You can have repeats for the letters and for the digits, so each of these will be ordered lists. $26^3 \cdot 10^4$

2. This is a permutation problem. With no restrictions, no additional rules will be used. $P(17, 3)$ is the solution.

3. There are three options: All math, all computer science, **or** all science. This means you will use the rule of sums to combine all outcomes together. Each outcome is a permutation problem - $P(8, 3)$, $P(12, 3)$, and $P(6, 3)$.

4. There are 8 letters and you're selecting all of them. Order matters since we're building a word. This is a permutation problem. The solution is $P(8, 8)$.

5. There are three outcomes: A subset with 1 item, or a subset with 3 items, or a subset with 5 items. Therefore, we will use the rule of sums. For each of these outcomes, we don't care about order. We are going to select 1 item from {1, 2, 3, 4, 5, 6 } ($C(6, 1)$), or we are going to select 3 items from {1, 2, 3, 4, 5, 6 } ($C(6, 3)$), or we are going to select 5 items from {1, 2, 3, 4, 5, 6 } ($C(6, 5)$).

6. You are drawing two cards... face up "and" face down, so the rule of products will be used. There are four different aces available in a deck, and four different 10's. We're only selecting one card for each outcome, but this is essentially a permutation problem. If we have $P(4, 1)$ for aces and $P(4, 1)$ for 10's, then the result is $4 \cdot 4$.

7. This is a combinations problem. There are 17 people to choose from, and we're selecting 3. There is no significance to the order they're selected, and we're not selecting for different roles, so the solution is $C(17, 3)$.

8. This is a combination problem, we don't care about order. There are 12 oranges, and we're selecting three. $C(12, 3)$.

9. When we're building a sequence of letters or numbers, generally order matters. 123 is a different number from 321. So, in this case, order matters. We can have repeats because we're selecting from the pool of {0, 1, 2}. We are selecting 12 items, so the result will be 3^{12}.

10. For a binary sequence, our option pool is $\{0, 1\}$. We need exactly six 1's, so we figure out a *position* for these items first: $C(8, 6)$. After that, there are only two spots where the remaining two 0's can go: $C(2, 2)$. Since we need *this outcome* "and" *that outcome*, we will use the rule of products: $C(8, 6) \cdot C(2, 2)$.

11. This is an unordered list. We can represent it as a binary sequence. There are 3 types of fruits, so we need 2 separators. There are spots for 20 pieces of fruit, and we add in the separators: $20 + 2 = 22$ total "spots". The result will be $C(22, 20)$

12.

n	0	1	2	3	4
a_n	2	5	8	11	14
$\triangle_n^1$	3	3	3	3	...

$$s_n = s_0 + \sum_{k=0}^{n-1} \triangle_k$$

$$s_n = 2 + \sum_{k=0}^{n-1} 3$$

$$s_n = 2 + 3n$$

Cheat sheet

The Rule of Product When you see the "and" keyword, you should be multiplying the different outcomes.

The Rule of Sum When you see the "or" keyword, you should be adding the different outcomes.

The Rule of Sum with Overlap When you're choosing *this* "or" *that*, and this & that have overlap, you calculate it with $this + that - overlap$.

The Rule of Complements If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$.

Structures

Type	Repeats allowed?	Order matters?	Formula
Ordered list of length r	yes	yes	n^r
Unordered list of length r	yes	no	$C(r+n-1, r)$
Permutations of length r	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Sets of length r	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$

Binary sequences

The number of binary sequences with r 1's and $n - r$ 0's is $C(n, r)$ or $C(n, n-r)$.

Fruits: When building a problem where you have t types of fruits and you are selecting p pieces of fruit, then the amount of separators, the 1's, is $t + 1$, and the amount of fruits, the 0's, is p, and the total amount of spots you're filling is $n = p + t + 1$, then you can pass in $r = t + 1$ (the # of 1's) or $r = p$ (the # of 0's).