Answer Key

Combinatorics: Intro and Structures

Combinatorics are counting problems, asking how many different combinations or outcomes there can be given some input(s), and when selecting a certain amount of items. These problems differ in how you solve them based on the type of structure.

Types of structures
These are the four types of structures we will be working with in this section.

	Repeats allowed?	**Order matters?**	**Formula**
Permutation n items to choose from Select r items	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Set n items to choose from Select r items	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$
Ordered list n items to choose from Select r items	yes	yes	n^r
Unordered list n different types Select r items	yes	no	$C(n+r-1, n)$
Unordered list for Binary Strings r 1's $n-r$ 0's	yes	no	$C(n,r)$

The challenging part of combinatorics is **figuring out which structure** is described in a problem, and what values to plug in. In this part we will get acquainted with simple problems for each structure, and in future exercises we will look at rules needed for more sophisticated problems.

Ordered Lists

Order matters **Repeats allowed**

Given n options, select r items: There are n^r ways to choose.

Question 1

You are rolling two dice: a red die and a green die. How many results are possible?

- There are n total outcomes per one die:
- We are rolling r dice:
- Result, n^r:

Question 2

An arcade machine prompts you to enter your initials for the high-score table. You can enter any capital letter (A-Z means 26 options), or any number (0-9 means 10 options). You have 3 slots to enter letters. How many ways can you write in a name?

- There are n total options to choose from, for one "space":
- We have r spaces to fill:
- Result, n^r:

Permutations

Order matters **Repeats NOT allowed**

Given n options, select r items:

$$P(n, r) = \frac{n!}{(n-r)!}$$

ways to choose.

Question 3

In a class of 10 students, you are going to elect a President, Vice President, and Secretary. How many ways can you elect three students?

- There are n total students to choose from:
- We are selecting for r different roles:
- Result, $P(n, r)$:

Question 4

A class of 10 students need to line up for lunch. How many ways are there for all students to line up?

- There are n total students to line up from:
- We are going to line up r students: *(Hint: same as n)*
- Result, $P(n, r)$:

Sets

Order DOESN'T matter **Repeats NOT allowed**

Given n options, select r items:

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

ways to choose.

Question 5

In a class of 10 students, you are going to choose 3 students to be on the holiday planning committee. How many ways can you elect three students for the committee? *Here, the students all fulfill the same role, so there is no difference between choosing the first student, second student, or third student.*

- There are n total students to choose from:
- We are selecting r students for the role:
- Result, $C(n,r)$:

Question 6

There is a box of marbles that has 6 green and 4 blue. How many ways are there to select 2 green marbles out of the box?

- There are n total green marbles:
- We are going to pick out r green marbles:
- Result, $C(n,r)$:

Unordered Lists

Order DOESN'T matter **Repeats allowed**

Binary String: Given r 1's and $n - r$ 0's, there are $C(n, r)$ ways to choose.

Question 7

How many ways are there to build a binary string (1's and 0's only) of length 4 that has only one 1?

- There are r 1's:
- There are $n - r$ 0's:
- n is:
- Result, $C(n, r)$:

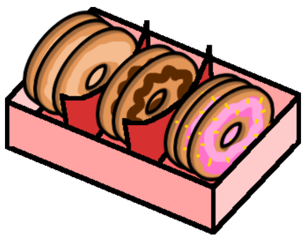
Order DOESN'T matter **Repeats allowed**

Selection way: Given n types of items and r items to choose, there are $C(n+r-1, r)$ ways to choose.

Binary String way: Given r 1's and $n-r$ 0's, there are $C(n, r)$ ways to choose.

Question 8

At a donut shop, we are going to buy 6 donuts. The store sells 3 types of donuts.



How many ways are there to buy 6 donuts?

- Selection way:
  - We are choosing r donuts:
  - There are n options for each donut:
  - Result, $C(n+r-1, r)$:
- Binary string way: We're going to consider 0's to represent the donuts.
  We're going to consider 1's to be separators *between each type* of donut, meaning that if there are 3 types of donuts, there are 2 *separators*.
  - There are r 1's:
  - There are $n-r$ 0's:
  - n is:
  - Result, $C(n, r)$:

0.1 Answer Key

1. $n = 6$, $r = 2$, $6^2 = 36$
2. $n = 36$, $r = 3$, $36^3 = 46,656$
3. $n = 10$, $r = 3$, $P(10, 3) = 720$
4. $n = 10$, $r = 10$, $P(10, 10) = 3,628,800$
5. $n = 10$, $r = 3$, $C(10, 3) = 120$
6. $n = 6$, $r = 2$, $C(6, 2) = 15$
7. $r = 1$, $n - r = 3$, $n = 4$, $C(4, 1) = 4$
8. $r = 6$, $n = 3$, $C(8, 6) = 28$ or $C(8, 2) = 28$