Answer Key

Combinatorics: The Rule of Sums

Review: Structures

Structures

	Repeats allowed?	Order matters?	Formula
Permutation n items to choose from Select r items	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Set n items to choose from Select r items	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$
Ordered list n items to choose from Select r items	yes	yes	n^r
Unordered list n different types Select r items	yes	no	$C(n+r-1,n)$
Unordered list for Binary Strings r 1's $n-r$ 0's	yes	no	$C(n,r)$

By Rachel Singh, last updated December 3, 2018

The Rule of Sums

The Rule of Sums
In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, **it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.** [a]

[a] From https://en.wikipedia.org/wiki/Rule_of_sum

Question 1
An arcade machine prompts you to enter your initials for the high-score table. You can enter any capital letter (A-Z means 26 options), or any number (0-9 means 10 options). You have 3 slots to enter letters. How many ways can you enter 3 characters, if you do all letters **or** all numbers?

- Scenario 1:
  - How many options to choose from (n):
  - How many characters to choose (r):
  - Result 1 (n^r):
- Scenario 2:
  - How many options to choose from (n):
  - How many characters to choose (r):
  - Result 2 (n^r):
- Result: (Result 1 + Result 2):

Question 2
In a class of 12 students, there are 5 CS students, 3 IT students, and 4 Math students. You are going to elect a President, Vice President, and Secretary. How many ways can you elect three students, if each role is filled by students with the same major?

- Does order matter? *Is this a set {Rai, Rose, Rahaf}, or would it be an ordered group (Rai, Rose, Rahaf) ?*
- Are repetitions allowed? *Can somebody be chosen more than once?*
- What structure is this?
- Scenario 1: Each role is filled only by CS students.
- Scenario 2: Each role is filled only by IT students.
- Scenario 3: Each role is filled only by Math students.
- Result: (Scenario 1 OR Scenario 2 OR Scenario 3)

Question 3

You are drawing a card from a deck of 52 cards. How many ways are there to...

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠
A ♢	2 ♢	3 ♢	4 ♢	5 ♢	6 ♢	7 ♢	8 ♢	9 ♢	10 ♢	J ♢	Q ♢	K ♢
A ♡	2 ♡	3 ♡	4 ♡	5 ♡	6 ♡	7 ♡	8 ♡	9 ♡	10 ♡	J ♡	Q ♡	K ♡

a. Get one red card?

b. Get one Jack OR a Queen?

c. Get one club OR a diamond?

d. Get one face card OR an Ace?

e. Get two Jacks OR two Queens?

The rule of sums with overlap
If the list to count can be split into two pieces of size x and y, and the pieces have z objects in common, then the original list has $x+y-z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B. [a]

[a] From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

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Question 4

You are drawing a card from a deck of 52 cards. How many ways are there to...

A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣
A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠
A ♢	2 ♢	3 ♢	4 ♢	5 ♢	6 ♢	7 ♢	8 ♢	9 ♢	10 ♢	J ♢	Q ♢	K ♢
A ♡	2 ♡	3 ♡	4 ♡	5 ♡	6 ♡	7 ♡	8 ♡	9 ♡	10 ♡	J ♡	Q ♡	K ♡

1. Get a Diamond card?

2. Get a an Ace card?

3. Get an Ace card OR a Diamond card? *Use the Rule of Sums with Overlap*

Question 5
At a muffin shop, you are going to buy two muffins - one for you, and one for your friend. There are chocolate muffins, banana nut muffins, and pumpkin muffins available today. How many ways are there to buy two muffins where you and your friend both get the same kind?

Answer Key

1. Scenario 1: $n = 26$, $r = 3$, Result: 26^3
   Scenario 2: $n = 10$, $r = 3$, Result: 10^3
   Scenario 1 or Scenario 2: $26^3 + 10^3 = 17,576 + 1,000 = 18,576$

2. Order matters
   Repetition not allowed
   Permutation
   Scenario 1: $P(5, 3) = 60$
   Scenario 2: $P(3, 3) = 6$
   Scenario 3: $P(4, 3) = 24$
   Result: $60 + 6 + 24 = 90$

3. One red: 26
   One jack OR one queen: $4 + 4 = 8$
   One club OR one diamond: $13 + 13 = 26$
   One face or one ace: $12 + 4 = 16$
   Two jacks or two queens: $P(4, 2) + P(4, 2) = 12 + 12 = 24$

4. One diamond: 13
   One ace: 4
   Ace OR diamond: 13 + 4 - 1 = 16

5. 1: Chocolate and chocolate (1), 2: Pumpkin and pumpkin (1),
   3: Banana and banana (1).
   Result: $1 + 1 + 1 = 3$