

Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group.**

Names:

Combinatorics: Combinations

Combinations and Permutations

Permutations

A permutation is a type of structure that describes counting when **order matters** and **repetitions are not allowed**.

A permutation is written as $P(n, r)$ where n is the amount of items we have to choose from, and r is the amount of items we are selecting. The formula for this is:

$$P(n, r) = \frac{n!}{(n - r)!}$$

Where $n!$ is n -factorial. Also note that $0! = 1$.

Combinations

In mathematics, a combination is selection of items from a collection, such that (unlike permutations) the order of selection does not matter. (...) The number of r -combinations from a given set of n elements is often denoted in elementary combinatorics texts by $C(n, r)$.^a

For a combination of length r from a set of n elements:

$$C(n, r) = \frac{n!}{(n - r)! \cdot r!}$$

(Note that the book uses r and Wikipedia uses k .)

^aFrom <https://en.wikipedia.org/wiki/Combinations>

Question 1

How many ways can you rearrange the letters in the word “DOG”?

Repetition allowed? Yes No

Order matters? Yes No

Structure?

Solve:

DOG

DGO

GOD

GDO

ODG

OGD

Question 2

In how many ways can 10 children line up for lunch?

Repetition allowed? Yes No

Order matters? Yes No

Structure?

Solve:



The Rule of Sums:

If we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are $A + B$ ways to choose one of the actions. ^a

The Rule of Complements: If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$. ^b

The Rule of Products: If there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions. ^c

^aFrom https://en.wikipedia.org/wiki/Rule_of_sum

^bFrom Discrete Mathematics, Ensley and Crawley, page 390

^cFrom https://en.wikipedia.org/wiki/Rule_of_product

Question 3

In a class of 10 students, there are 3 IT majors and 7 CS majors. If 4 board positions had to be filled for the computer club, how many ways would there be to fill the positions with the given constraints?



- a. No constraints - any student can be on the board.

- b. There must be exactly 2 IT students and 2 CS students on the board.

- c. There must be *at least* 2 IT students on the board.

Question 4

Suppose we are going to receive a shipment of 50 games on floppy disk for our vintage game store. Each box of 50 generally has 3 defective floppies. For the shipment, we are going to select 5 games to feature in a display.

- a. How many total good floppies are there?

- b. How many total bad floppies are there?

- c. How many ways could we choose 5 games to feature?

- d. How many ways contain *no* defective floppies?

- e. Using the Rule of Products, determine how many ways that contain all 3 defective floppies?

- f. Using the Rule of Sums and the Rule of Products, determine how many ways contain *at least one* defective floppy.

- g. Using the Rule of Complements, determine how many ways contain *at least one* defective floppy. The answer for (f) and (g) should match.

Question 5

There's a bargain bin that has 5 PC games, 3 Playstation games, and 8 Xbox games.

- a. How many total games are there?

- b. If you're grabbing 4 games to buy, you don't care about the order that you pull the games out. How many ways can 4 games be selected?

- c. How many ways can you select 4 games that are all for the same console? (*Hint: This means the Playstation games don't get counted. Also, which Rule are you going to use to solve this?*)

- d. How many selections of 4 games are there, where you have 2 for one platform, and 2 for another platform? (*Hint: What Rules apply here?*)