Answer Key

Combinatorics: The Rule of Products

Review: Structures

Permutation formula

	Repeats allowed?	Order matters?	Formula
Permutation n items to choose from Select r items	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Set n items to choose from Select r items	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$
Ordered list n items to choose from Select r items	yes	yes	n^r
Unordered list r 1's $n-r$ 0's	yes	no	$C(n,r)$

By Rachel Singh, last updated December 3, 2018

Review: The Rule of Sums

The Rule of Sums

In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, **it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.** [a]

[a]From https://en.wikipedia.org/wiki/Rule_of_sum

The rule of sums with overlap

If the list to count can be split into two pieces of size x and y, and the pieces have z objects in common, then the original list has $x+y-z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B. [a]

[a]From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

The Rule of Products

The rule of products

In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, **it is the idea that if there are a ways of doing something and b ways of doing another thing, then there are $a \cdot b$ ways of performing both actions.** [a]

[a] From https://en.wikipedia.org/wiki/Rule_of_product

For these questions, make sure to identify whether you will need the Rule of Sums, the Rule of Products, or both.

Question 1

At a company, each employee ID begins with two upper-case letters (26 options, A-Z) and then three digits (10 options, 0-9). How many ways are there to build an employee ID?

- Ways to select the letters:
- Ways to select the numbers:
- Building the full ID, letters AND numbers:

Question 2

At a company, each employee ID begins with two upper-case letters (26 options, A-Z) and then three digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). How many ways are there to build an employee ID, if the digits must all be EVEN or all be ODD? (Consider 0 an even number for this question)

Question 3

You're going with a group of 20 to the zoo (21 including you). There are 5 parents and 15 kids. Your car can hold 4 (other) people. How many ways can you take a group of 4 people with you, if there is at least two parents in your car?

Scenarios: 2 parents, 3 parents, 4 parents

Question 4

At a conference, there are 10 talks about web tech, 3 talks about development tools, and 7 talks about best practices. How many ways can you see 3 talks if...

a. There's no restriction on what topics?

b. You have to see talks on one of each topic?

c. You have to see talks on all the same topics?

The Rule of Complements

The rule of complements
If there are x objects, and y of those objects have a particular property, then the number of those objects that do **not** have that particular property is $x - y$. [a]

[a] From Discrete Mathematics, Ensley and Crawley, page 390

Question 5
There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a committee of 3 people. How many ways are there to elect a committee where AT LEAST ONE member is a math student?

Answer key

1. Letters: 26^2
   Numbers: 10^3
   Result: $26^2 \cdot 10^3 = 676,000$

2. Evens: $26^2 \cdot 5^3 = 84,500$
   Odds: $26^2 \cdot 5^3 = 84,500$
   Result: $84,500 + 84,500 = 169,000$

3. Two parents, two kids: $C(5,2) \cdot C(15,2) = 1,050$
   Three parents, one kid: $C(5,3) \cdot C(15,1) = 150$
   Four parents, no kids: $C(5,4) = 5$
   Result: $1050 + 150 + 5 = 1,205$

4. Conference, 10 web, 3 tools, 7 best practices, see 3 talks...
   This is a bad question, because it isn't clear if you can see the same talk multiple times. Assuming duplicates not allowed and order doesn't matter...

   a. $C(20,3) = 1,140$

   b. $C(10,1) \cdot C(3,1) \cdot C(7,1) = 210$

   c. $C(10,3) + C(3,3) + C(7,3) = 156$

5. No restrictions: $C(20,3) = 1140$
   NO MATH students: $C(14,3) = 364$
   At least one math student: $1140 - 364 = 776$