Answer Key

Combinatorics: Review

Overview

The main concepts in this section are the four Structures (Permutations, Sets, Ordered Lists, and Unordered Lists), the Rule of Sums, the Rule of Sums with Overlap, the Rule of Products, and the Rule of Complements.

Given the following cheatsheet, are you able to solve the following problems?

Cheat Sheet

	Repeats allowed?	Order matters?	Formula
Permutation n items to choose from Select r items	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Set n items to choose from Select r items	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$
Ordered list n items to choose from Select r items	yes	yes	n^r
Unordered list n different types Select r items	yes	no	$C(n+r-1,r)$
Unordered list for Binary Strings r 1's $n-r$ 0's	yes	no	$C(n,r)$

The Rule of Sums: If there are A ways of doing one thing, and B ways of doing another, then the total amount of ways you can do "A or B" is $A + B$.

The Rule of Sums with Overlap: If there are A ways of doing one thing, and B ways of doing another, but there is an overlap of C, then the total amount of ways you can do "A or B" is $A + B - C$.

The Rule of Products: If there are A ways of doing one thing, and B ways of doing another, then the total amount of ways you can do "A and B" is $A \cdot B$.

The Rule of Complements: If there are C total ways to perform some action, and A total ways meet some criteria, then the total ways that *don't* meet that criteria is $A' = C - A$. In other words, $A + A' = C$

Level 1: No rules

Question 1
In a class of 25 students, how many ways can a 5-person committee be formed?
Structure:

Solution:

Question 2
In a class of 20 students, you want to elect a President, Vice President, and Secretary. How many ways can you elect people to these positions?
Structure:

Solution:

Question 3
How many ways are there to select 4 cards from a standard deck of 52 cards?
Structure:

Solution:

Question 4
If we want to build a binary string with 5 characters that has exactly two 1's, how many ways are there to build this string?
Structure:

Solution:

Question 5
If there are 4 types of muffins available, and you're buying two muffins for yourself, how many ways can you select muffins?
Structure:

Solution:

Question 6
If there are 4 types of muffins available, and you're buying one muffin for you and one muffin for a friend, how many ways can you select muffins?
Structure:

Solution:

Level 2: Using rules

Question 7
At an adoption center, there are 15 dogs available for adoption. There are 5 Labrador Retrievers, 3 Corgis, and 7 Maltese dogs.

Structure:

If you're able to adopt 3 dogs, how many ways can you select dogs if...

a. There's no restriction.

b. You must get one of each breed of dog.

c. All 3 dogs you're adopting must be of the same breed.

d. You cannot adopt any Labradors due to size.

e. You must adopt *at least one* Labrador.

Question 8
In C++, variable names can contain lower case letters (`a-z`, 26 options), upper case letters (`A-Z`, 26 options), and numbers (`0-9`, 10 options), and underscores (_, 1 option). How many ways are there to create a variable name of length 5 if...

a. There's no restriction.

b. The variable name *cannot* begin with a number, but it can begin with any other character and numbers can occur in all following characters.

c. The variable name has letters, numbers, and underscores, but it can only use all UPPERCASE or all lowercase.

Question 9
100 people are running in a race. A person can win 1st place, 2nd place, or 3rd place. There are...

- 15 people younger than 20
- 25 people between 20 - 30
- 20 people who are 30 - 40
- 30 people who are 40 - 50
- 10 people who are 50+

How many ways are there for there to be 1st, 2nd, and 3rd place if...

a. There's no restriction.

b. 1st, 2nd, and 3rd place are all held by people from the same age group.

c. One person under 30, one person between 30 - 50, and one person over 50 wins a place.

d. Someone under 20 wins 1st place (but not 2nd or 3rd), OR someone under 20 wins 2nd and 3rd place (but not 1st).

Answer key

1. Set: $C(25, 5) = 53,130$
2. Permutation: $P(20, 3) = 6,840$
3. Permutation: $P(52, 4) = 6,497,400$
4. Unordered List (Binary string): $r = 2$, $n - r = 3$, $n = 5$, $C(5, 2) = 10$
   11000 10100 10010 10001 01100
   01010 01001 00110 00101 00011
5. Unordered List: 4 types of muffins, buying 2 muffins for yourself.
   Using the binary string method: $r = 3$, $n - r = 2$, $n = 5$, $C(5, 3) = 10$
   Using the other form, $n = 4$, $r = 2 = C(4 + 2 - 1, 2) = C(5, 2) = 10$ {a, a}, {a, b}, {a, c}, {a, d}, {b, b}, {b, c}, {b, d}, {c, c}, {c, d}, {d, d}
6. Ordered List: $4^2 = 16$
7. Sets: 15 dogs; 5 labs, 3 corgis, 7 maltese.
   a. $C(15, 3) = 455$
   b. Rule of Products: $C(5, 1) \cdot C(3, 1) \cdot C(7, 1) = 5 * 3 * 7 = 105$
   c. Rule of Sums: $C(5, 3) + C(3, 3) + C(7, 3) = 46$
   d. $C(10, 3) = 120$
   e. Rule of Complements: $C(15, 3) - C(10, 3) = 335$
8. Ordered List: 26 lower case, 26 upper case, 10 numbers, 1 underscore.
   a. $63^5 = 992,436,543$
   b. First character: 53 options.
      Subsequent characters: 63 options.
      Result: $53 \cdot 63^4 = 834,906,933$
   c. Only uppercase: 26+10+1 options; 37^5
      Only lowercase: 26+10+1 options; 37^5
      Result: All uppercase OR all lowercase: $37^5 + 37^5 = 138,687,914$

9. Permutation: 15 *[0,20)*, 25 *[20, 30)*, 20 *[30,40)*, 30 *[40,50)*, 10 *[50, ...)*

   a. $P(100, 3) = 970,200$

   b. $P(15, 3) + P(25, 3) + P(20, 3) + P(30, 3) + P(10, 3) = 48,450$

   c. $P(40, 1) \cdot P(50, 1) \cdot P(50, 1) = 100,000$

   d. $P(15, 1) \cdot P(85, 2) + P(85, 1) \cdot P(15, 2) = 124,950$