Combinatorics: Intro and Structures

Types of structures

	Repeats allowed?	Order matters?	Formula
Permutation n items to choose from Select r items	no	yes	$P(n,r) = \frac{n!}{(n-r)!}$
Set n items to choose from Select r items	no	no	$C(n,r) = \frac{n!}{r!(n-r)!}$
Ordered list n items to choose from Select r items	yes	yes	n^r
Unordered list n different types Select r items	yes	no	$C(n+r-1,n)$
Unordered list for Binary Strings r 1's $n-r$ 0's	yes	no	$C(n,r)$

The Rule of Sums
In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, **it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.** [a]

[a] From https://en.wikipedia.org/wiki/Rule_of_sum

The rule of sums with overlap
If the list to count can be split into two pieces of size x and y, and the pieces have z objects in common, then the original list has $x + y - z$ entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B. [a]

[a] From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

Homework

1. Rai wants to adopt two cats. At the adoption center, there are 10 available cats: 6 girls and 4 boys. If Rai wants to get two cats of the same gender, how many ways can he select his cats?

2. You're going to roll two dice. How many ways are there for you to get all even or all odd numbers?

3. You're going to roll two dice. How many ways are there for you to get all even or numbers greater than 4?

4. How many different ways are there to rearrange the letters D, O, U, G if it either begins and ends with a consonant, OR begins and ends with a vowel?

Answer key

1. Scenario 1: Get two girl cats: $C(6,2)$
   Scenario 2: Get two boy cats: $C(4,2)$
   Result: $C(6,2) + C(4,2) = 15 + 6 = 21$

2. Scenario 1: All even: $3^2 = 9$
   { (2, 2) (4, 2) (6, 2) (2, 4) (4, 4) (6, 4) (2, 6) (4, 6) (6, 6) }
   Scenario 2: All odd: $3^2 = 9$
   { (1, 1) (3, 1) (5, 1) (1, 3) (3, 3) (5, 3) (1, 5) (3, 5) (5, 5) }

   Result: $9 + 9 = 18$

   Written out: {
   (2, 2) (4, 2) (6, 2) (2, 4) (4, 4) (6, 4) (2, 6) (4, 6) (6, 6)
   (1, 1) (3, 1) (5, 1) (1, 3) (3, 3) (5, 3) (1, 5) (3, 5) (5, 5)
   }

3. Scenario 1: All even: $3^2 = 9$
   { (2, 2) (4, 2) (6, 2) (2, 4) (4, 4) (6, 4) (2, 6) (4, 6) (6, 6) }
   Scenario 2: Numbers greater than 4: $2^2 = 4$
   { (5, 5) (6, 5) (5, 6), (6, 6) }

   Overlap: even numbers greater than 4: just (6,6). $= 1$ outcome

   Result: $9 + 4 - 1 = 12$

   Written out: {
   (2, 2) (4, 2) (6, 2) (2, 4) (4, 4) (6, 4) (2, 6) (4, 6) (6, 6)
   (5, 5) (6, 5) (5, 6)
   }
   The duplicate between both would be (6,6), so it has only been written once.

4. Think of this problem as writing out the "slots" where we fill in letters...

   _____ _____ _____ _____

   For the first option, we will put the consonants at the beginning and end. For the first letter we're placing, we have two options:

   P(2,1)

   _____ _____ _____ _____

   D/G

   For the last option, we only have one choice after choosing the first...

   P(2,1) P(1,1)

   _____ _____ _____ _____

   D/G

   And for the middle options, we are selecting two letters from the two remaining (O, U)...

P(2,1)	P(2,2)	P(1,1)
First consonant	Middle letters	Last consonant
(Two options)	(Two, select 2)	(One option)

   These values get multiplied for the result.
   $2 \cdot 2 \cdot 1 \cdot 1 = 4$ ways to have the consonants on the outside. If we write them out: { (D, O, U, G), (G, O, U, D), (D, U, O, G), (G, U, O, D) }

   It will be the same calculation for vowels on the outside; also 4.

   Therefore, the result is $4 + 4 = 8$. Written out:

   {
   (D, O, U, G), (G, O, U, D), (D, U, O, G), (G, U, O, D)
   (O, D, G, U), (U, D, G, O), (O, G, D, U), (U, G, D, O)
   }