# The Rule of Sums

Written by Rachel Singh, last updated Nov 15, 2018

# About

In some combinatorics problems, we're interested in multiple outcomes: outcome A OR outcome B OR outcome C… So how do you calculate these problems?

# Topics

1. The Rule of Sums

2. The Rule of Sums with Overlap

3. Basic practice problems

# The Rule of Sums

# 1. The Rule of Sums

In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.

From https://en.wikipedia.org/wiki/Rule of sum

Notes

**Permutation** — **P(n,r)**
- Repeats? no
- Order? yes

**Sets** — **C(n,r)**
- Repeats? no
- Order? no

**Ordered List** — **$n^r$**
- Repeats? yes
- Order? yes

**Unordered List**
C(n,r)* Or C(n+r-1, r)
- Repeats? yes
- Order? no

# 1. The Rule of Sums

In combinatorics, the rule of sum or addition principle is a basic counting principle. Stated simply, it is the idea that if we have A ways of doing something and B ways of doing another thing and we can not do both at the same time, then there are A + B ways to choose one of the actions.

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This rule works specifically if you have two separate events that can't overlap…

- Get a "heads" on a coin flip, OR get a "tails" on a coin flip ✓
- Get a "1" on a die roll, OR get a "2" on a die roll ✓
- Draw a "Heart" card, OR draw a "Diamond" card ✓

- Draw a "Heart" card, OR draw an "Ace" card ✘ **These can overlap!**

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# 1. The Rule of Sums

**Example:** Let's say you're flipping a Quarter and a Penny. How many ways can you get both heads OR both tails?

Notes

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- **Order?** **yes**

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- **Repeats?** **yes**
- **Order?** **no**

# 1. The Rule of Sums

**Example:** Let's say you're flipping a Quarter and a Penny. How many ways can you get both heads OR both tails?

Scenario 1: Both heads – 1 outcome

Scenario 2: Both tails – 1 outcome

Result: 1 + 1 = 2 outcomes

All scenarios:
(H, H)
(H, T)
(T, H)
(T, T)

Notes

| | |
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# The Rule of Sums with Overlap

# 2. The Rule of Sums with Overlap

If the list to count can be split into two pieces of size x and y, and the pieces have z objects in common, then the original list has x+y−z entries. In terms of sets, we can write this as $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for all sets A and B.

From Discrete Math Mathematical Reasoning and Proofs with Puzzles, Patterns and Games, by Ensley and Crawley

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This way is useful for if we want to find out the ways to get "this event" OR "that event" to occur, but they **could** overlap.

Because there's overlap, if we use the first sum rule, we would end up **double-counting** that duplicate result.

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# 2. The Rule of Sums with Overlap

**Example:** You're rolling a die. How many ways could you get an even number OR a number greater than 3?

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# 2. The Rule of Sums with Overlap

**Example:** You're rolling a die. How many ways could you get an <u>even number</u> OR <u>a number greater than 3?</u>

Scenario 1: Even numbers = 3 outcomes

Scenario 2: Greater than three = 3 outcomes

Overlap: Even numbers greater than three = 2

Result: 3 + 3 – 2 = 4 outcomes

| A: | B: |
|---|---|
| 2 | 4 |
| 4 | 5 |
| 6 | 6 |

Result:
2
4
5
6

## Notes

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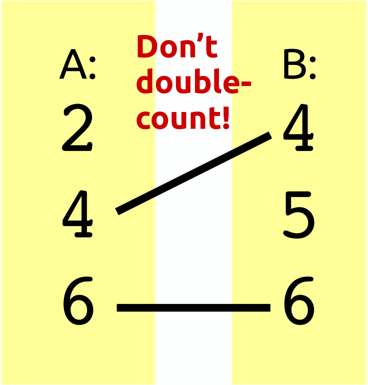


Result:
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# Practice Problems

# 3. Practice Problems

**Example:** There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a committee of 3 students.

How many ways are there to have the committee be made up of all the same major?

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| | | |
|---|---|---|
| Scenario 1: All CS students | = C(10, 3) | = 120 |
| Scenario 2: All IT students | = C(4, 3) | = 4 |
| Scenario 3: All math students | = C(6, 3) | = 20 |

**Result: 120 + 4 + 20 = 144**

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# 3. Practice Problems

**Example:** You're drawing 3 cards from a standard deck of 52 cards. How many ways can you get all Aces OR all Diamonds?

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# 3. Practice Problems

**Example:** You're drawing 3 cards from a standard deck of 52 cards. How many ways can you get all Aces OR all Diamonds?

Scenario 1: All aces

There are 4 aces in a deck, so **P(4, 3) = 24**

Scenario 2: All diamonds

There are 13 diamonds in a deck, so **P(13, 3) = 1716**

Overlap: An Ace of Diamonds

There is one of these cards in the deck, do **P(1, 1) = 1**

**Result:** **P(4, 3) + P(13, 3) – P(1, 1)** **= 24 + 1716 – 1**

**= 1739**

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# Conclusion

Make sure you're taking time to properly identify each structure before solving the problems…

Is repetition allowed?

Does order matter?