Combinations

Written by Rachel Singh, last updated Sep 30, 2018

About

Last time we talked about Ordered Lists, Unordered Lists, and Permutations. This time we will talk about Combinations, where we consider the outcomes as being *sets* – order doesn't matter, but repetition is not allowed.

Topics

1. Combinations
2. Revisiting Rules

Combinations

1. Combinations

A combination is written as C**(n, r)**. For Combination problems, you need two pieces of information:

- *n*, the amount of items we have to select from
- *r*, the amount of items that we're selecting.

The formula for C(n, r) is:

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

1. Combinations

With a permutation, order matters.
With a combination, order doesn't matter.

For both of these, there cannot be repetitions.

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

An example of a problem with permutations would be where items are ranked, or given different properties.

"How many ways can you elect a president, vice president, and secretary?"

Whereas with a combination, position isn't given any meaning.

"How many ways can three people be put on a committee?"

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is *n* ?
- What is *r* ?
- What's the answer?

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is n? **100**
- What is r? **5**
- What's the answer?

$$C(100,5)=\frac{100!}{5!(100-5)!} = \frac{100 \times 99 \times 98 \times \ldots \times 5 \times 4 \times 3 \times 2 \times 1}{(5 \times 4 \times 3 \times 2 \times 1) \times (95 \times 94 \times \ldots \times 5 \times 4 \times 3 \times 2 \times 1)}$$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is n ? **100**
- What is r ? **5**
- What's the answer?

Can cancel out 5x4x3x2x1 in the numerator and 5! in the denominator…

$$= \frac{100 \times 99 \times 98 \times \ldots \times 5 \times 4 \times 3 \times 2 \times 1}{(5 \times 4 \times 3 \times 2 \times 1) \times (95 \times 94 \times \ldots \times 5 \times 4 \times 3 \times 2 \times 1)}$$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r) = \frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is n ? **100**
- What is r ? **5**
- What's the answer?

95 x 94 x … x 7 x 6 can be canceled out.

$$= \frac{100 \times 99 \times 98 \times \ldots \times 8 \times 7 \times 6}{(95 \times 94 \times \ldots \times 5 \times 4 \times 3 \times 2 \times 1)}$$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r) = \frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is *n* ? **100**
- What is *r* ? **5**
- What's the answer?

Then it can be simplified. $= \dfrac{100 \times 99 \times 98 \times 97 \times 96}{5 \times 4 \times 3 \times 2 \times 1} = \dfrac{9034502400}{120}$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r) = \frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

- What is *n* ? **100**
- What is *r* ? **5**
- What's the answer?

Then it can be simplified. $= \frac{9034502400}{120} = 75287520$

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

You can also use Wolfram Alpha to solve it.

WolframAlpha computational knowledge engine.

C(100,5)

Result:

$$\frac{9034502400}{120} = 75287520$$

75 287 520

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

1. Combinations

Example 2 from the textbook: How many five-person committees can be formed from the 100-member U.S. Senate?

For an exam, this is the important part:

- What is *n* ? **100**
- What is *r* ? **5**

$$C(100,5)=\frac{100!}{5!(100-5)!}$$

and the final numerical value is generally less important:

75 287 520

Notes

C(n, r):
n is # of potential items
r is # of selections

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

P(n, r):
n is # of potential items
r is # of selections

$$P(n,r)=\frac{n!}{(n-r)!}$$

Revisiting Rules

2. Revisiting Rules

Remember that the Rule of Sums is for when we want to find the amount of combinations given

resultA OR resultB

and the Rule of Products is for when we want to find the amount of combinations given

resultA AND resultB

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

For some problems, we might not be able to find the result with a single Combination or a single Permutation; we will have to solve multiple Combination problems and then **combine** them.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

a. No constraints on members.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

a. No constraints on members.

n = **18** r = **5**

C(18, 5) = 8,568

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

b. The committee contains *exactly* three women.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

b. The committee contains *exactly* three women.

Here, we know 3 members will be women, <u>**and**</u> 2 will be men. We can separate this out:

$$\mathbf{C(10,\ 3)} \quad \times \quad \mathbf{C(8,\ 2)}$$

- Total women (10)
- Women to select (3)
- Total men (8)
- Men to select (2)

Notes

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

b. The committee contains *exactly* three women.

Here, we know 3 members will be women, **and** 2 will be men. We can separate this out:

C(10, 3) x C(8, 2)

= 120 x 28

= 3,360 different ways to build this committee.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

Now we need to look at the options:

- Exactly three women and two men, **<u>OR</u>**
- Exactly four women and one man, **<u>OR</u>**
- Exactly five women and no men.

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

- Exactly three women and two men, C(10,3) x C(8,2) **OR**
- Exactly four women and one man, C(10,4) x C(8,1) **OR**
- Exactly five women and no men. C(10,5)

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

- Exactly three women and two men, C(10,3) x C(8,2) **OR**
- Exactly four women and one man, C(10,4) x C(8,1) **OR**
- Exactly five women and no men. C(10,5)

C(10, 3) x C(8, 2) **+** **C(10, 4) x C(8, 1)** **+** **C(10, 5)**

AND **OR** **AND** **OR**

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

C(10, 3) x C(8, 2) **+** **C(10, 4) x C(8, 1)** **+** **C(10, 5)**

AND **OR** **AND** **OR**

Note that for an exam, THIS is the important part!
But we can also calculate the final number…

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

2. Revisiting Rules

Example 4 from the book: A club of ten women and eight men is forming a five-person steering committee. How many possible committees are there given the following constraint:

c. The committee contains *at least* three women.

C(10, 3) x C(8, 2)	**+**	**C(10, 4) x C(8, 1)**	**+**	**C(10, 5)**
AND	**OR**	**AND**	**OR**	

C(10, 3) * C(8, 2) + C(10, 4) * C(8, 1) + C(10, 5)

Result:

5292

Notes

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$P(n,r)=\frac{n!}{(n-r)!}$$

Either one thing *or* another thing: **a + b**

This or that, without duplicates: **a + b – c**

Doing one thing *and* another thing: **a x b**

Conclusion

Now we've covered the basic structures used in the counting problems we will encounter in this chapter.