The Rule of Products

Written by Rachel Singh, last updated Nov 26, 2018

About

Last time, we talked about the Rule of Sums, where we want to see how many ways Event 1 OR Event 2 could occur.

We use the Rule of Products when we're interested in how many ways we can get Event 1 to occur, AND Event 2 to occur.

Topics

1. The Rule of Products
2. The Rule of Complements
3. Basic practice problems

The Rule of Products

1. The Rule of Products

In combinatorics, the rule of product or multiplication principle is a basic counting principle (a.k.a. the fundamental principle of counting). Stated simply, it is the idea that if there are **a** ways of doing something and **b** ways of doing another thing, then there are **a · b** ways of performing both actions.

From https://en.wikipedia.org/wiki/Rule_of_product

Notes

Permutation	**P(n,r)**
- Repeats?	no
- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
- Order?	no

Ordered List	n^r
- Repeats?	yes
- Order?	yes

Unordered List	**C(n,r)* Or C(n+r-1, r)**
- Repeats?	yes
- Order?	no

1. The Rule of Products

Example: You are flipping a coin four times. How many ways can the first two flipf match each other, and the second two flips match each other?

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- Repeats?	yes
- Order?	no

1. The Rule of Products

Example: You are flipping a coin four times. How many ways can the first two flipf match each other, and the second two flips match each other?

First flip:	2 options
Second flip (must match):	1 option
Third flip:	2 options
Fourth flip (must match):	1 option

Notes

Permutation	**P(n,r)**
- Repeats?	no
- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
- Order?	no

Ordered List	n^r
- Repeats?	yes
- Order?	yes

Unordered List

C(n,r)* Or C(n+r-1, r)

- Repeats?	yes
- Order?	no

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Second flip (must match):	1 option
Third flip:	2 options
Fourth flip (must match):	1 option

Result = 2x1x2x1 = 4

(H, H, H, H) (H, H, T, T) (T, T, H, H) (T, T, T, T)

Notes

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- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
- Order?	no

Ordered List	**n^r**
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- Order?	yes

Unordered List	**C(n,r)* Or C(n+r-1, r)**
- Repeats?	yes
- Order?	no

1. The Rule of Products

Sometimes, you might get a problem where you will need to use both the Rule of Sums AND the Rule of Products…

Example: You are rolling a dice 4 times. How many ways can you have either the 1^{st} and 4^{th} roll match, OR the 2^{nd}, 3^{rd}, and 4^{th} roll match?

Notes

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- Repeats?	**no**
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1. The Rule of Products

Sometimes, you might get a problem where you will need to use both the Rule of Sums AND the Rule of Products…

Example: You are rolling a dice 4 times. How many ways can you have either the 1^{st} and 4^{th} roll match, OR the 2^{nd}, 3^{rd}, and 4^{th} roll match?

Scenario 1, 1^{st} and 4^{th} roll match

First roll:	6 options
Fourth roll (must match):	1 option
Second roll (no restriction):	6 options
Third roll (no restriction):	6 options

= 6 x 1 x 6 x 6 = 216 outcomes.

Notes

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- Repeats?	no
- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
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Ordered List	**n^r**
- Repeats?	yes
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1. The Rule of Products

Sometimes, you might get a problem where you will need to use both the Rule of Sums AND the Rule of Products…

Example: You are rolling a dice 4 times. How many ways can you have either the 1^{st} and 4^{th} roll match, OR the 2^{nd}, 3^{rd}, and 4^{th} roll match?

Scenario 2, 2^{nd}, 3^{rd}, and 4^{th} roll match

First roll (no restriction):	6 options
Second roll (no restriction):	6 options
Third roll (must match):	1 option
Fourth roll (must match):	1 option

= 6 x 6 x 1 x 1 = 36 outcomes.

Notes

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- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
- Order?	no

Ordered List	n^r
- Repeats?	yes
- Order?	yes

Unordered List	C(n,r)* Or C(n+r-1, r)
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- Order?	no

1. The Rule of Products

Sometimes, you might get a problem where you will need to use both the Rule of Sums AND the Rule of Products…

Example: You are rolling a dice 4 times. How many ways can you have either the 1^{st} and 4^{th} roll match, OR the 2^{nd}, 3^{rd}, and 4^{th} roll match?

Scenario 1: 216 outcomes

Scenario 2: 36 outcomes

Result: 216 + 36 = 252 ways

Notes

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The Rule of Complements

2. The Rule of Complements

If there are **x** objects, and **y** of those objects have a particular property, then the number of those objects that **do not** have that particular property is **x - y**.

From Discrete Mathematics, Ensley and Crawley, page 390

In some problems, it would be a lengthy calculation to figure it out – but, if you do the opposite instead (how many ways to NOT meet this criteria…), you can solve it much quicker.

With the problems you'll see, usually the keyword that tips you off to using the Rule of Complements is "**at least**".

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- Order?	no

2. The Rule of Complements

Example: You are flipping a coin four times. How many ways are there to get **at least one** heads? (1 head, 2 heads, 3 heads, 4 heads)

All outcomes:

H,H,H,H	H,H,H,T	H,H,T,H	H,H,T,T	H,T,H,H	H,T,H,T	H,T,T,H	H,T,T,T
T,H,H,H	T,H,H,T	T,H,T,H	T,H,T,T	T,T,H,H	T,T,H,T	T,T,T,H	T,T,T,T

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Rule of Sums method:

1 heads scenarios:
First,
Second,
Third,
Fourth

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T,H,H,H	T,H,H,T	T,H,T,H	T,H,T,T	T,T,H,H	T,T,H,T	T,T,T,H	T,T,T,T

Rule of Sums method:

1 heads scenarios:
First,
Second,
Third,
Fourth

2 heads scenarios:
First, second,
First, third,
First, fourth,
Second, third,
Second, fourth,
Third, fourth

Notes

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- Order?	yes

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Rule of Sums method:

1 heads scenarios:
First,
Second,
Third,
Fourth

2 heads scenarios:
First, second,
First, third,
First, fourth,
Second, third,
Second, fourth,
Third, fourth

3 heads scenarios:
First, second, third,
First, second, fourth,
First, third, fourth,
Second, third, fourth

Notes

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Rule of Sums method:

1 heads scenarios:
First,
Second,
Third,
Fourth

2 heads scenarios:
First, second,
First, third,
First, fourth,
Second, third,
Second, fourth,
Third, fourth

3 heads scenarios:
First, second, third,
First, second, fourth,
First, third, fourth,
Second, third, fourth

4 heads secenarios:
First, second, third, fourth

= 4 + 6 + 4 + 1 = 15 ways

Notes

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Rule of Complements method:

How many ways to select, no restriction?
$2^4 = 16$

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How many ways to select, no restriction?
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How many ways to select, NO HEADS?
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Rule of Complements method:

How many ways to select, no restriction?
$2^4 = 16$

How many ways to select, NO HEADS?
1

NO HEADS	+	at least one heads	=	total
1	+	x	=	16

At least one heads: 16 – 1 = 15 ways

Notes

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Practice Problems

3. Practice Problems

Example: There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a President, Vice President, and Secretary.

How many ways are there to elect a committee, if there is no restriction on who can be President and Vice President, but the Secretary MUST be a Math major?

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In this case, we need to choose the **restricted option** first:

Secretary role:	P(6, 1)	(6 math majors)
President role:	P(19, 1)	(19 people remaining)
Vice President role:	P(18, 1)	(18 people remaining)

Pres/VP can also be written as P(19, 2).

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P(6, 1)	x	P(19, 2)
Elect a Secretary	AND	Elect a Pres/VP

= 6 x 342

= 2052

Notes

Permutation	P(n,r)
- Repeats?	no
- Order?	yes

Sets	C(n,r)
- Repeats?	no
- Order?	no

Ordered List	n^r
- Repeats?	yes
- Order?	yes

Unordered List	C(n,r)* Or C(n+r-1, r)
- Repeats?	yes
- Order?	no

3. Practice Problems

Example: There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a President, Vice President, and Secretary.

How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

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How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Long way:

1 role is math:
Math pres
Math VP
Math sec

= 3276

P(6,1) x P(14, 2) = 1092

P(6,1) x P(14, 2) = 1092

P(6,1) x P(14, 2) = 1092

P(6,1) x P(14, 2) because we're selecting ONE math major (out of 6) and TWO CS/IT (out of 14)

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
Math 6	Math 5	Math 4
IT/CS 14	IT/CS 13	IT/CS 12

Notes

Permutation **P(n,r)**
- Repeats? no
- Order? yes

Sets **C(n,r)**
- Repeats? no
- Order? no

Ordered List **n^r**
- Repeats? yes
- Order? yes

Unordered List
C(n,r)* Or **C(n+r-1, r)**
- Repeats? yes
- Order? no

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How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Long way:

1 role is math:
Math pres
Math VP
Math sec

= 3276

2 roles are math:
Math pres, VP — P(6,2) x P(14, 1) = 420
Math pres, sec — P(6,2) x P(14, 1) = 420
Math VP, sec — P(6,2) x P(14, 1) = 420

= 1260

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
Math 6	Math 5	Math 4
IT/CS 14	IT/CS 13	IT/CS 12

P(6,2) x P(14, 1) because we're selecting TWO math majors (out of 6) and ONE CS/IT (out of 14)

Notes

Permutation — **P(n,r)**
- Repeats? no
- Order? yes

Sets — **C(n,r)**
- Repeats? no
- Order? no

Ordered List — **n^r**
- Repeats? yes
- Order? yes

Unordered List
C(n,r)* Or **C(n+r-1, r)**
- Repeats? yes
- Order? no

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Example: There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a President, Vice President, and Secretary.

How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Long way:

1 role is math:
Math pres
Math VP
Math sec

= 3276

2 roles are math:
Math pres, VP
Math pres, sec
Math VP, sec

= 1260

3 roles are math:
Math pres, VP, sec

= 120

P(6,3) = 120

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
Math 6	Math 5	Math 4
IT/CS 14	IT/CS 13	IT/CS 12

P(6,3) because we're selecting THREE math majors (out of 6)

Notes

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- Repeats?	no
- Order?	yes

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- Repeats?	no
- Order?	no

Ordered List	**n^r**
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Unordered List	**C(n,r)* Or C(n+r-1, r)**
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How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Long way:

1 role is math:
Math pres
Math VP
Math sec

= 3276

2 roles are math:
Math pres, VP
Math pres, sec
Math VP, sec

= 1260

3 roles are math:
Math pres, VP, sec

= 120

= 3276 + 1260 + 120 **= 4656 ways**

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
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Ordered List	**n^r**
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- Order?	yes

Unordered List

C(n,r)* Or **C(n+r-1, r)**

- Repeats? yes
- Order? no

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How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Rule of complements:

No restrictions on roles:
P(20, 3) = 6840

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
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3. Practice Problems

Example: There are 20 students in a class: 10 CS students, 4 IT students, and 6 math students. We're going to elect a President, Vice President, and Secretary.

How many ways are there to elect the three positions, with AT LEAST ONE ROLE filled by a math student?

Rule of complements:

No restrictions on roles:
P(20, 3) = 6840

NO roles with math:
P(14, 3) = 2184

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
Math 6	Math 5	Math 4
IT/CS 14	IT/CS 13	IT/CS 12

Notes

Permutation — **P(n,r)**
- Repeats? no
- Order? yes

Sets — **C(n,r)**
- Repeats? no
- Order? no

Ordered List — **n^r**
- Repeats? yes
- Order? yes

Unordered List
C(n,r)* Or **C(n+r-1, r)**
- Repeats? yes
- Order? no

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NO MATH	**+**	**at least 1 math**	**= total**
2184	**+**	**x**	**= 6840**

At least 1 math = 6840 – 2184 = 4656 ways

PRES	VP	SEC
Math 6	IT/CS 14	IT/CS 13
IT/CS 14	Math 6	IT/CS 13
IT/CS 14	IT/CS 13	Math 6
Math 6	Math 5	IT/CS 14
Math 6	IT/CS 14	Math 5
IT/CS 14	Math 6	Math 5
Math 6	Math 5	Math 4
IT/CS 14	IT/CS 13	IT/CS 12

Notes

Permutation	**P(n,r)**
- Repeats?	no
- Order?	yes

Sets	**C(n,r)**
- Repeats?	no
- Order?	no

Ordered List	**n^r**
- Repeats?	yes
- Order?	yes

Unordered List

C(n,r)* Or C(n+r-1, r)

- Repeats?	yes
- Order?	no

Conclusion

In addition to identifying the structure / which formula to use for the problem, you will also need to pay attention to whether a problem will require the Rule of Sums AND/OR the Rule of Products.