CS 191 Exam 1 Review, Fall 2018

Grading Scheme

Every question will have the same point breakdown, but they have different weights. Points will be allocated as:

Nothing written	Something attempted, but incorrect	Partially correct, but multiple errors	Mostly correct, with one or two errors	Perfect; correct answer and notation
0	1	2	3	4

Cheatsheet provided

Sets:

Intersection	Union	Difference	Prime
$\cap$	$\cup$	$-$	$'$ or bar ($\bar{X}$)

Common sets:

$\mathbb{N}$, the set of natural numbers	$\mathbb{Z}$, the set of integers
$\mathbb{Q}$, the set of rational numbers	$\mathbb{R}$, the set of all real numbers

Cartesian product of A and B: The cartesian product of two sets $A \times B$ is the combination of all elements from A and all elements from B in ordered pairs.

Power set of C: The power set of C is defined as $\wp(C) = \{S : S \subseteq C\}$. The power set of C is the set of all possible subsets of C, including the empty set.

Partition of D: A partition of D is a set $S = \{S_1, S_2, S_3, ...\}$, where S_i is one part. Each part is a set, and no two sets have anything in common. All elements of the original set D must be represented in these parts. $S_1 \cap S_2 \cap ... \cap S_3 = \emptyset$ and $S_1 \cup S_2 \cup ... \cup S_3 = D$

Proposition: A propositional statement is a statement that is unambiguously true or false. A propositional variable can be assigned to a statement to save space. Compound propositions can be formed using logic operators.

Logic operators: And: $\wedge$, Or: $\vee$, Not: $\neg$

Implication $p \rightarrow q$: An implication is an "if, then" statement, where you have a hypothesis and a conclusion. It is written $p \rightarrow q$ as a general form, where p is the hypothesis and q is the conclusion. The negation of an implication is NOT an implication.

Predicate $P(x)$: A predicate is a logical function that takes some input (x) and returns a propositional value: true or false. The output of this function depends on the input.

Domain: The domain given is a set of all possible inputs x that can be plugged into the predicate $P(x)$.

Quantifiers: You can use a quantifier to specify when a predicate is true. For all: $\forall$, There exists (at least one): $\exists$.

Questions

1.1: Sets

Question 1

Find the results, given the following sets:

$U = \{a, b, c, d, e, f, g\}$ $A = \{a, b, c\}$ $B = \{d, e, f\}$

$C = \{a, c, e, f, g\}$ $D = \{f, g\}$

a. $A \cap B =$

b. $A \cap C =$

c. $A \cup B =$

d. $B \cup D =$

e. $(A \cup B)' =$

f. $U - (A \cup B) =$

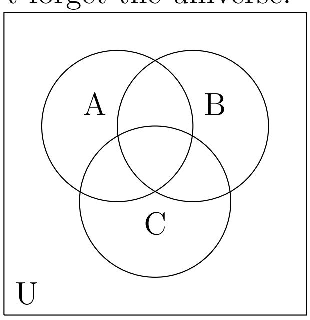
g. $C - D =$

h. $(C - D) \cap A =$

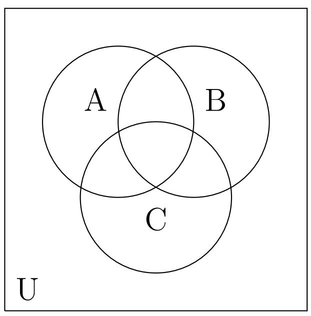
i. $(B \cup D) \cap C =$

Question 2

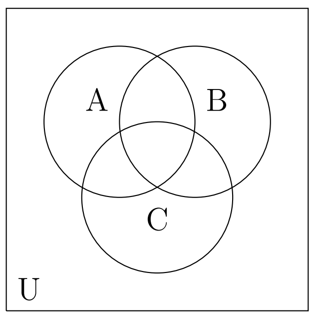
Shade in the Venn diagrams for the given statements.
Don't forget the universe.



a. $U - B$



b. $A' - B$



c. $(B - A) \cup C$

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Question 3

Given the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and sets $A = \{2, 4\}$, $B = \{1, 2, 8\}$, and $C = \{1, 2, 5, 6, 10\}$, find each of the following:

a. $A \times B$

b. $(A \times B) - (A \times A)$

c. $\wp(B)$

d. $\wp(B \cap C)$

e. $\wp(B) - \wp(B \cap C)$

From Discrete Mathematics, Ensley & Crawley, Chapter 3.2 exercise 1, page 208

Question 4

Given the following set, create partitions that match each of the following criteria.

$$A = \{1, 2, 3, 4\}$$

a. All parts of the partition are the same size:

b. No two parts are the same size:

c. There are the fewest possible amount of parts:

d. There are the most possible amount of parts:

1.2: Propositions

Question 5

Finish the truth table for the following expression: $(p \land q) \lor (p \land \neg r)$

p	q	r
T	**T**	**T**
T	**T**	F
T	F	**T**
T	F	F
F	**T**	**T**
F	**T**	F
F	F	**T**
F	F	F

Question 6

Given three propositional variables p, q, and r, write a compound statement that will meet the following criteria. Use parenthesis to explicitly define the order of operation. [1]

a. p and r are true, but not q.

b. r and one other statement are true, but not all three.

c. exactly two statements are true, but not only one or all three.

Question 7

Translate the following statements into propositional logic using the given variables.

D : We can dance S : We can sing
L : We can leave your friends behind F : Your friends dance
M : Your friends are friends of mine

a. We can dance and sing.

b. We can dance and we can leave your friends behind.

c. Your friends don't dance, and your friends are not friends of mine.

d. We can dance and sing, or, we can dance and your friends dance.

e. We can dance or we can sing, and we can't leave your friends behind.

f. Either we can dance or we can sing, but not both.

g. We cannot dance and we cannot sing, but your friends are friends of mine.

[1] Logical operator precedence: 1. $\neg$, 2. $\wedge$, 3. $\vee$, 4. $\rightarrow$

1.3: Implications

Question 8
Given the following propositional variables:

y: you go $\qquad$ i: I go

Write out each of the following in English:

a. Original implication, $y \to i$:

b. Negation, $y \wedge \neg i$:

c. Inverse, $\neg y \to \neg i$:

d. Converse, $i \to y$:

e. Contrapositive, $\neg i \to \neg y$:

1.5, 1.6: Quantifiers and Predicates

Question 9
Given the following predicate, define a domain that makes the quantified statement either $true$ or $false$.

a. $P(n)$ is the predicate, "n ends with the number 5."
   Quantified statement: $\forall n \in D, P(n)$ is true.
   $D =$

b. $Q(n)$ is the predicate, "n is divisible by 4."
   Quantified statement: $\forall n \in D, Q(n)$ is false.
   $D =$

Question 10

a. Given the domain $D = \{2, 3, 4, 5, 6\}$, Translate the following statement into a quantified statement using predicate logic:
"For every element x that is a member of the domain D, x is odd."

Write "x is odd" in English, not symbolically.

b. Write the negation of the statement from (a). Do not use the symbol $\neg$ for the final form, and write the predicate in English.

c. Which statement is true: (a) or (b)?

Question 11

Let D be the domain of all people. Given the following predicates, translate the following English statements into logical statements, using the $\neg, \wedge, \vee, \rightarrow, \exists, \forall$ symbols. Make sure each statement has either $\exists$ or $\forall$.

$W(x)$ is "x is a wizard"	$K(x)$ is "x is a knight"	$V(x)$ is "x is a valkyrie"
$M(x)$ is "x can use magic"	$S(x)$ is "x can use swords"	

a. For all people x, if x is a wizard, then x can use magic.

b. For all people x, if x is a knight, then x cannot use magic.

c. There exists a person x that is a knight, and x can use magic.

d. For all people x, if x can use swords, then x is a knight or a valkyrie.

e. There is some person x who can use magic and swords, and is a valkyrie.

Answer Key

1. a. $A \cap B = \quad \emptyset$
   b. $A \cap C = \quad \{a, c\}$
   c. $A \cup B = \quad \{a, b, c, d, e, f\}$
   d. $B \cup D = \quad \{d, e, f, g\}$
   e. $(A \cup B)' = \quad \{g\}$
   f. $U - (A \cup B) = \quad \{g\}$
   g. $C - D = \quad \{a, c, e\}$
   h. $(C - D) \cap A = \quad \{a, c\}$
   i. $(B \cup D) \cap C = \quad \{e, f, g\}$

2. a. $U - B$
   b. $A' - B$
   c. $(B - A) \cup C$

3. a. $A \times B$
   $\{(2, 1), (2, 2), (2, 8), (4, 1), (4, 2), (4, 8)\}$
   b. $(A \times B) - (A \times A)$
   $\{(2, 1), (2, 8), (4, 1), (4, 2), (4, 8)\}$
   c. $\wp(B)$
   $\{\emptyset, \{1\}, \{2\}, \{8\}, \{1, 2\}, \{1, 8\}, \{2, 8\}, \{1, 2, 8\}\}$
   d. $\wp(B \cap C)$
   $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$
   e. $\wp(B) - \wp(B \cap C)$
   $\{\{8\}, \{1, 8\}, \{2, 8\}, \{1, 2, 8\}\}$

4. a. All parts of the partition are the same size:
   Multiple answers. Example: $S = \{\{1\}, \{2\}, \{3\}, \{4\}\}$, or $S = \{\{1, 2\}, \{3, 4\}\}$.
   b. No two parts are the same size:
   Multiple answers. Example: $S = \{\{1\}, \{2, 3, 4\}\}$
   c. There are the fewest possible amount of parts:
   $S = \{\{1, 2, 3, 4\}\}$
   d. There are the most possible amount of parts:
   $S = \{\{1\}, \{2\}, \{3\}, \{4\}\}$

By Rachel Singh, last updated September 6, 2018

5. Truth table:

p	q	r	$(p \wedge q) \vee (p \wedge \neg r)$
T	**T**	**T**	**T**
T	**T**	F	**T**
T	F	**T**	F
T	F	F	**T**
F	**T**	**T**	F
F	**T**	F	F
F	F	**T**	F
F	F	F	F

6. Writing statements

   a. $p \wedge \neg q \wedge r$

   b. $r \wedge (q \vee p) \wedge \neg(p \wedge q \wedge r)$

   c. $(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r)$

7. a. We can dance and sing. $D \wedge S$

   b. We can dance and we can leave your friends behind. $D \wedge L$

   c. Your friends don't dance, and your friends are not friends of mine $\neg F \wedge \neg M$

   d. We can dance and sing, or, we can dance and your friends dance. $(D \wedge S) \vee (D \wedge F)$

   e. We can dance or we can sing, and we can't leave your friends behind. $(D \vee S) \wedge \neg L$

   f. Either we can dance or we can sing, but not both. $(D \vee S) \wedge \neg(D \wedge S)$

   g. We cannot dance and we cannot sing, but your friends are friends of mine. $\neg D \wedge \neg S \wedge M$

8. a. $P(n)$ is the predicate, "n ends with the number 5."
   Quantified statement: $\forall n \in D, P(n)$ is true.
   Example: $D = \{ \quad 5, 15, 25, 35, 45, 55, 105 \quad \}$

   b. $Q(n)$ is the predicate, "n is divisible by 4."
   Quantified statement: $\forall n \in D, Q(n)$ is false.
   Example:$D = \{ \quad 4, 12, 16, 40, 44, 80, 120 \quad \}$

9. a. Original implication, $y \to i$: If you go then I go.

   b. Negation, $y \wedge \neg i$: You go and I do not go.

   c. Inverse, $\neg y \to \neg i$: If you don't go, then I don't go.

   d. Converse, $i \to y$: If I go, then you go.

e. Contrapositive, $\neg i \rightarrow \neg y$: If I don't go then you don't go.

10. a. *"For every element x that is a member of the domain D, x is odd."*
    $\forall x \in D, x$ is divisible by 2.

    b. Write the negation of the statement from (a).
    $\exists x \in D, x$ is not divisible by 2.

    c. Which statement is true: (a) or (b)?
    The negation is true.

11. a. For all people x, if x is a wizard, then x can use magic.
    $\forall x \in D, W(x) \rightarrow M(x)$

    b. For all people x, if x is a knight, then x cannot use magic.
    $\forall x \in D, K(x) \rightarrow \neg M(x)$

    c. There exists a person x that is a knight, and x can use magic.
    $\exists x \in D, K(x) \wedge M(x)$

    d. For all people x, if x can use swords, then x is a knight or a valkyrie.
    $\forall x \in D, S(x) \rightarrow (K(x) \vee V(x))$

    e. There is some person x who can use magic and swords, and is a valkyrie.
    $\exists x \in D, M(x) \wedge S(x) \wedge V(x)$

By Rachel Singh, last updated September 6, 2018