Functions and Relations: Exam Review

Cheat sheet

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1. A function f is **onto** if every element of the codomain is mapped to by at least one element of the domain.
2. A function f is **one-to-one** if every element of the codomain is mapped to by at most one element of the domain.
3. A function f is **invertible** if its inverse f^{-1} as also a function.

Reference: Relation properties

1. A relation R on A is **reflexive** if for all $a \in A$, $(a, a) \in R$
2. A relation R on A is **irreflexive** if for all $a \in A$, $(a, a) \notin R$
3. A relation R on A is **symmetric** if for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \in R$
4. A relation R on A is **antisymmetric** if for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \notin R$
5. A relation R on A is **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, it must also be the case that $(a, c) \in R$.

Functions and Function Properties

Be familiar with the following:

- How do you determine if a given graph is a function?
- How do you determine if a given function's inverse is a function?
- What does "onto" mean?
- What does "one-to-one" mean?
- What is the domain and codomain? (Make sure you label these when drawing diagrams.)
- Know the function notation for this chapter: $f : A \to B$ with a rule.

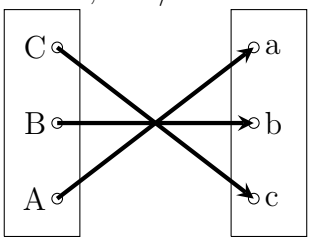
Practice problems

1. Diagram the following AND its inverse:
   Label your domain and codomain.
   $h : X \to Y$, where $X = \{2, 4, 6, 8\}$ and $Y = \{1, 3, 5, 7\}$.
   Rule: { (2, 7), (4, 3), (6, 5), (8, 1) }

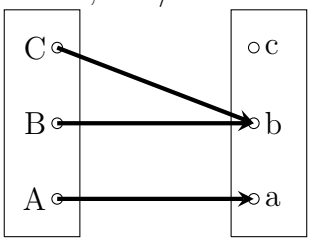
2. Diagram the following AND its inverse:
   Label your domain and codomain.
   $f : A \to A$, where $A = \{-3, -2, -1, 1, 2, 3\}$.
   Rule: $y = -x$.

3. Diagram the following AND its inverse:
   Just use 5 items from the domain and codomain.
   $g : \mathbb{Z} \to \mathbb{Z}$ where $y = x^2$.

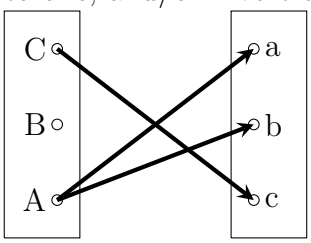
4. Identify whether the following is a function and whether it is onto, one-to-one, and/or invertible (i.e., is the inverse a function?)



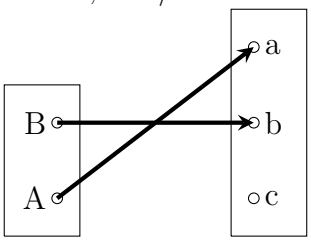
5. Identify whether the following is a function and whether it is onto, one-to-one, and/or invertible (i.e., is the inverse a function?)



6. Identify whether the following is a function and whether it is onto, one-to-one, and/or invertible (i.e., is the inverse a function?)



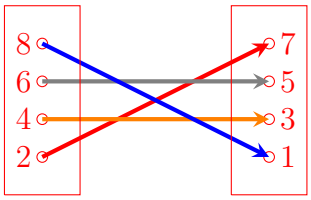
7. Identify whether the following is a function and whether it is onto, one-to-one, and/or invertible (i.e., is the inverse a function?)



Answer key

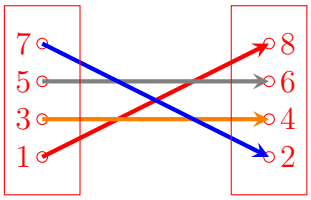
1. Rule: { (2, 7), (4, 3), (6, 5), (8, 1) }

Function



Domain Codomain

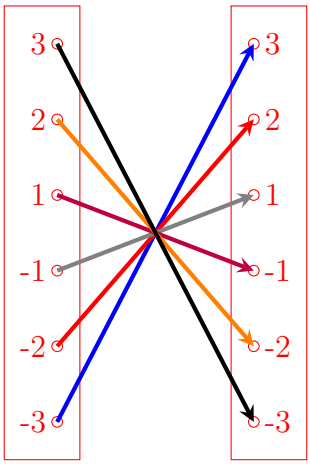
Inverse



Domain Codomain

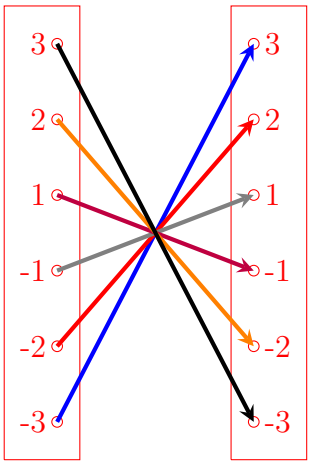
2. Rule: $y = -x$.

Function



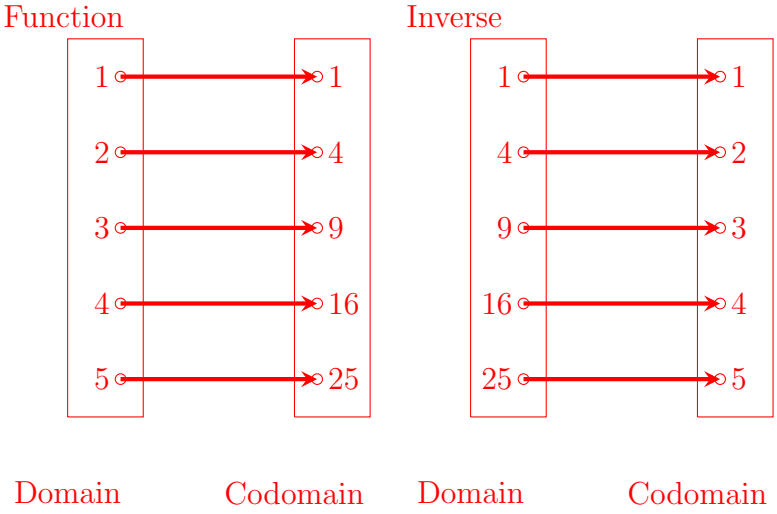
Domain Codomain

Inverse



Domain Codomain

3. $g : \mathbb{Z} \to \mathbb{Z}$ where $y = x^2$.



4. Function, one-to-one, onto, invertible, Inverse is a function.

5. Function, not onto, not one-to-one, Inverse is not a function.

6. Not a function, onto, one-to-one, Inverse is a function.

7. Function, not onto, one-to-one, Inverse is not a function.

Binary Relations

Be familiar with the following:

- A binary relation between sets A and B is a collection of ordered pairs of elements from A and B. The relation will be equal to, or a subset of, $A \times B$.
- If the domain and codomain in a relation are the same set, you can draw the relation as a graph instead of a function diagram.

Practice Problems

1. Diagram the relation $R_1 : A \to A$ given by the rule $A \times A$, where $A = \{1, 2, 3\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

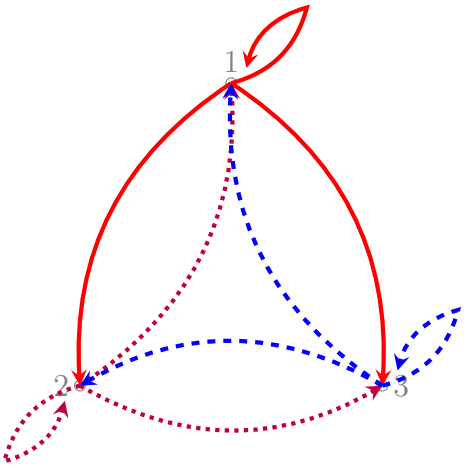
2. Diagram the relation $R_2 : A \to A$ given by the rule $\{(1, 1), (1, 2), (2, 3), (3, 2)\}$, where $A = \{1, 2, 3\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

3. Diagram the relation $R_3 = \{(a, b) \in \wp(B) \times \wp(B) : a \subseteq b\}$ where $B = \{1, 2\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

4. Diagram the relation $R_4 = \{(a, b) \in A \times A : \{(1, 1), (1, 2), (2, 1), (2, 3), (2, 4), (3, 2), (4, 2)\}\}$ where $A = \{1, 2, 3, 4\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

Answer key

1. $R_1 = \{(a,b) \in A \times A : A \times A\}$

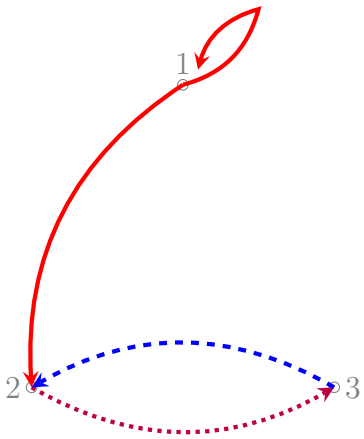


Reflexive - each node points back to itself.

Symmetric - each path has a reverse.

Transitive

2. $R_2 = \{(a,b) \in A \times A : \{(1,1),(1,2),(2,3),(3,2)\}\}$

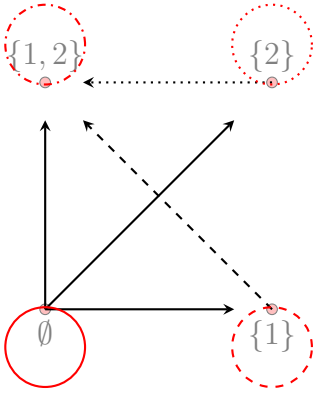


Neither Reflexive nor Irreflexive - Some nodes have loops.

Neither Symmetric nor Antisymmetric - Some paths go only one way.

Not Reflexive - Can't go from 1 to 3, but can go from $1 \to 2 \to 3$.

3. $R_3 = \{(a,b) \in \wp(B) \times \wp(B) : a \subseteq b\}$

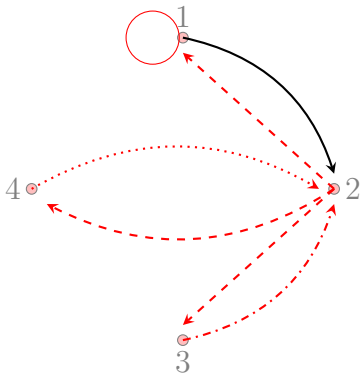


Reflexive

Antisymmetric

Transitive

4. $R_4 = \{(a,b) \in A \times A : \{(1,1),(1,2),(2,1),(2,3),(2,4),(3,2),(4,2)\}\}$



Neither Reflexive nor Irreflexive

Symmetric

Not transitive

Order Relations

Be familiar with the following:

- An order relation is a type of binary relation where the relationship between two points a, b *orders* them - one is greater than the other, or less than the other, and so on.

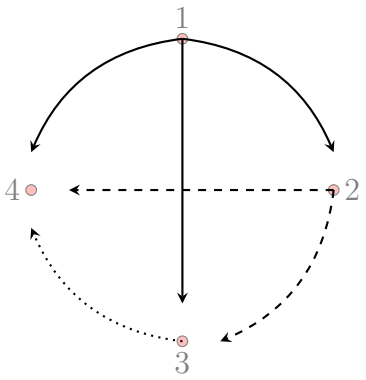
Practice Problems

1. Diagram the relation $R_5 = \{(a, b) \in A \times A : a < b\}$, where $A = \{1, 2, 3, 4\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not transitive

2. Diagram the relation $R_6 = \{(a, b) \in A \times A : a > b\}$, where $A = \{1, 2, 3, 4\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not transitive

Answer key

1. $R_5 = \{(a, b) \in A \times A : a < b\}$

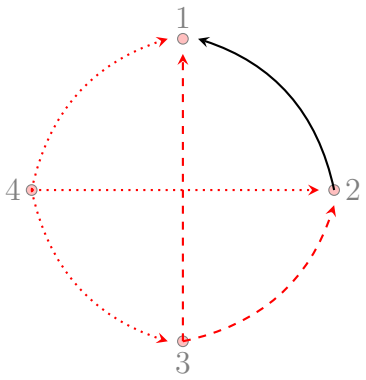


Irreflexive - No nodes loop back on themselves.

Antisymmetric - Each arrow goes only one way.

Transitive

2. $R_6 = \{(a, b) \in A \times A : a > b\}$



Irreflexive - No nodes loop back on themselves.

Antisymmetric - Each arrow goes only one way.

Transitive

Equivalence Relations

Be familiar with the following:

- An Equivalence Relation is a binary relation where the equivalence of two items (nodes) is defined by the rule. A relationship (arrow) is drawn between the two items if they're considered equivalent.

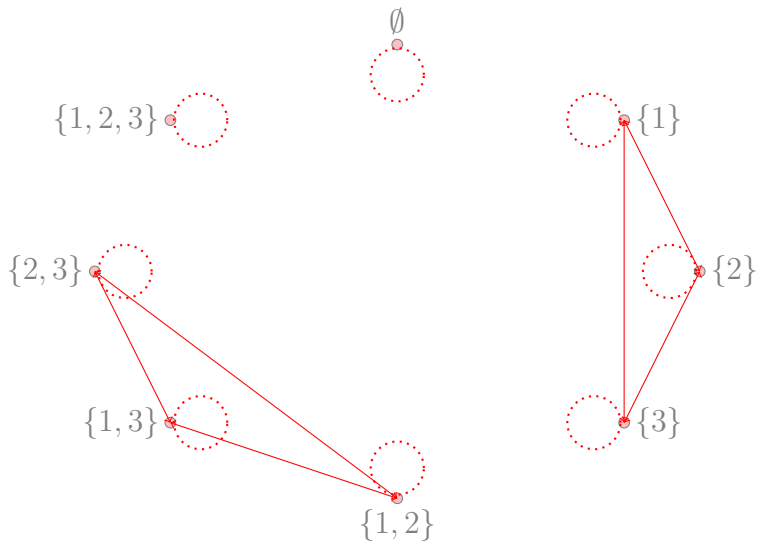
Practice Problems

1. Diagram the relation $R_7 = \{(A, B) \in \wp(S) \times \wp(S) : n(A) = n(B)\}$, where $S = \{1, 2, 3\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

2. Diagram the relation $R_8 = \{(A, B) \in \wp(S) \times \wp(S) : A \cap B = \emptyset\}$, where $S = \{1, 2, 3\}$, and identify the properties:
   ○ Reflexive ○ Irreflexive ○ Neither
   ○ Symmetric ○ Antisymmetric ○ Neither
   ○ Transitive ○ Not Transitive

Answer key

1. $R_7 = \{(A, B) \in \wp(S) \times \wp(S) : n(A) = n(B)\}$



2. $R_8 = \{(A, B) \in \wp(S) \times \wp(S) : A \cap B = \emptyset\}$