Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group**.

Names:

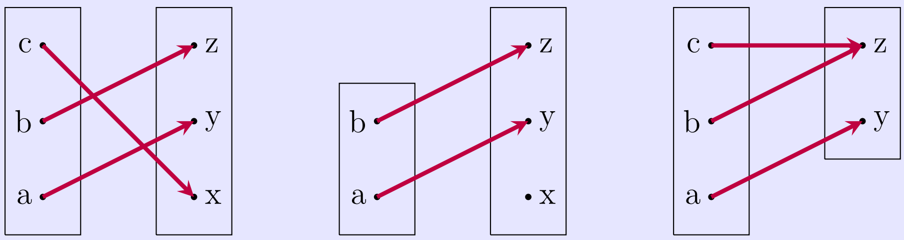
Functions and Relations: Relations and Relation Properties

Review: Functions

What is a Function?

A function is some mapping from an **input** from the **domain** to some **output** from the **codomain** (aka range). A valid function has exactly one mapping from every input to some output. If some input doesn't have an output, then it isn't a function.

A function is invertible (meaning, the inverse is also a function) only if it is both **onto** (every output in the codomain is mapped to by at least one input) and **one-to-one** (every output in the codomain is mapped to by at most one input).



Onto
One-to-one

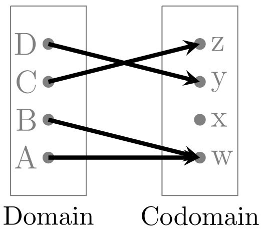
Not onto
One-to-one

Onto
Not one-to-one

Question 1
For each diagram, identify whether or not it is onto, one-to-one, and invertible.

For any graphs that are NOT onto or NOT one-to-one, explain why below.

a.

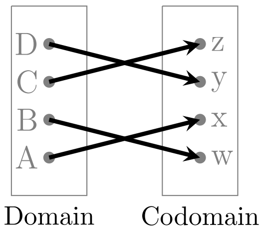


Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

b.

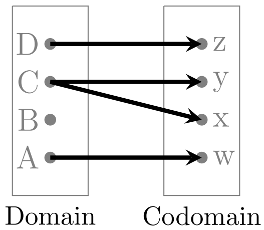


Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

c.

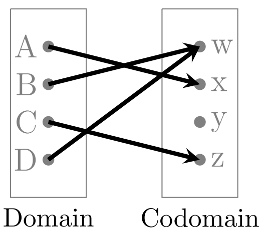


Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

d.

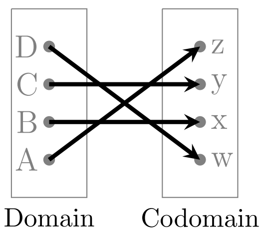


Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

e.

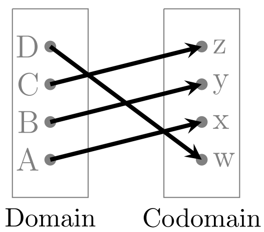


Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

f.



Onto?
○ Yes ○ No

One-to-one?
○ Yes ○ No

Invertible?
○ Yes ○ No

Question 2
Is the function $f : \mathbb{Z} \to \mathbb{Z}$ with the rule $f(x) = 3x + 1$ **one-to-one**, **onto**, **invertible**, or none?

From Discrete Mathematics, Ensley & Crawley, Chapter 4.3 exercise 3a, page 298

Question 3
For the function $f : \mathbb{N} \to \mathbb{N}$ with $f(n) = 2n$, draw a portion of the arrow diagram showing at least five elements of the domain and codomain.

From Discrete Mathematics, Ensley & Crawley, Chapter 4.1 exercise 1a, page 264

Relations

What is a Relation?
A relation is a mapping of a relationship between items. There is some **domain** X and some **codomain** Y, and the **rule** is the association between the two items. A relation may or may not be a function.

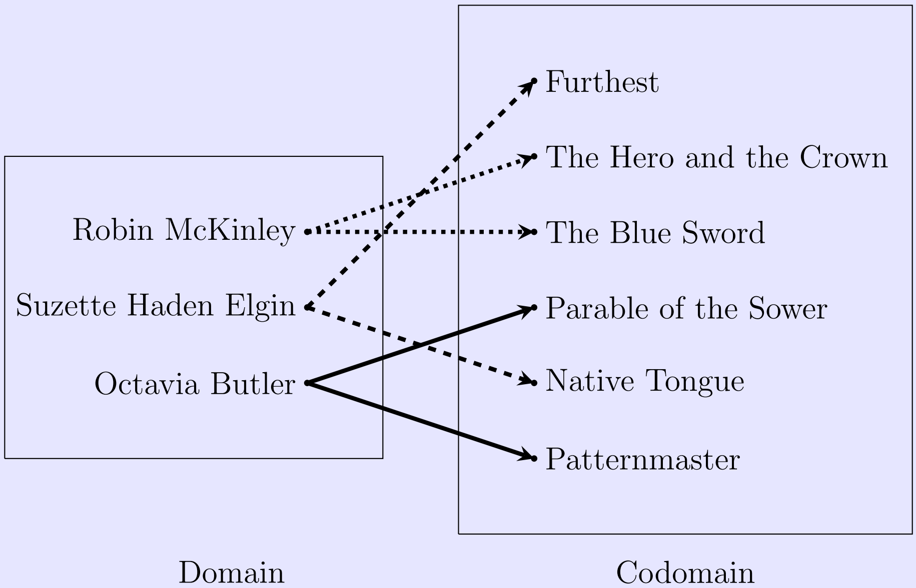
A = { Octavia Butler, Suzette Haden Elgin, Robin McKinley }

B = { Patternmaster, Native Tongue, Parable of the Sower, The Blue Sword, The Hero and the Crown, Furthest }

Table form:

Author	**Book**
Octavia Butler	Patternmaster
Suzette Haden Elgin	Native Tongue
Octavia Butler	Parable of the Sower
Robin McKinley	The Blue Sword
Robin McKinley	The Hero and the Crown
Suzette Haden Elgin	Furthest

Diagram form:



Question 4

Diagram the following relations.

a. Domain (students): { Levi, Cecelia, Ertan }
Codomain (courses): { C++ Coding, Discrete Math, Phonemes and You, Parolu Esperanton }
Rule: { (Levi, C++ Coding), (Cecelia, C++ Coding), (Ertan, Discrete Math), (Ertan, Phonemes and You), (Levi, Parolu Esperanton) }

Domain Codomain

b. Domain and Codomain: { 1, 2, 3, 4 }
Rule: Relationship (x, y) exists if $x < y$.
x and y are both retrieved from the same set (domain/codomain). There IS a relationship if the first number chosen is less than the second one.

Domain Codomain

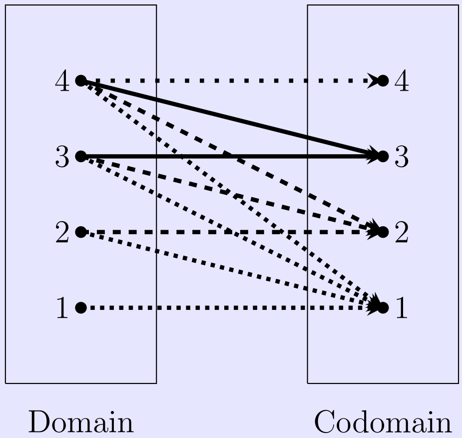
Another way to diagram relations

If a relation's doman and codomain are the same set, we can draw the set as a directional graph instead of as a diagram. In the directional graph, we draw an arrow between two nodes (points) **if the relationship exists**.

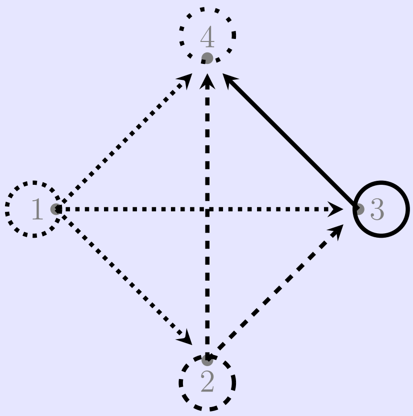
Example: A relation R with domain X and codomain X, and the given rule.

$X = \{1, 2, 3, 4\}$
Rule: Relationship (x, y) exists if $x \leq y$.

Normal diagram:



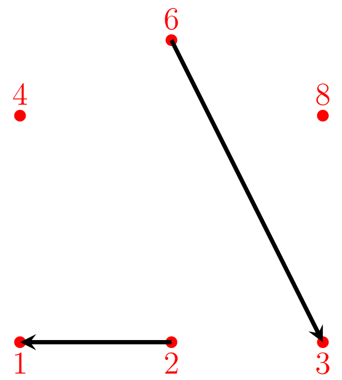
Graph:



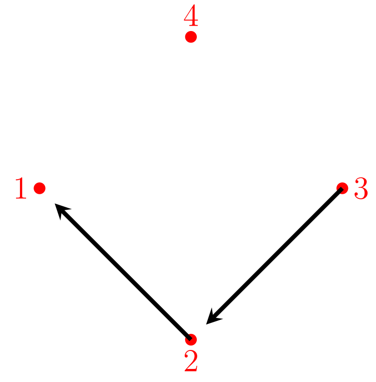
A circle signifies a relationship where $x = y$, e.g. $(3, 3)$.

Question 5
Finish the diagrams:

a. Domain and Codomain: { 1, 2, 3, 4, 6, 8 }
Rule: Relationship (x, y) exists if $x = 2y$.
The arrow should be pointing from x to y, if the relationship exists.



b. Domain and Codomain: { 1, 2, 3, 4 }
Rule: Relationship (x, y) exists if $x > y$.



Binary Relations

What is a Binary Relation?

A **Binary Relation** is what we've been covering - a relationship between elements of two sets X and Y, where the rule is some subset of $X \times Y$.

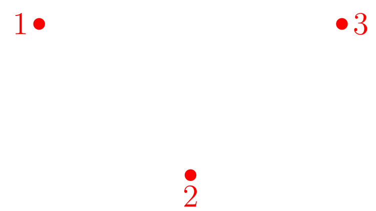
Given some relation $R : X \to Y$ (A relation R from X to Y), the relationship $(x, y) \in R$ (from x, pulled from X, to y, pulled from Y, exists in the relation R) if the rule given is met.

Question 6

Given $A = \{1, 2, 3\}$, solve each of the following.

a. What is the Cartesian product, $A \times A$?

b. Given the relation $R : A \to A$, and the rule $(x, y) \in R$ if $x + y$ is odd, diagram the relation:



c. List out the rule as a set. *e.g., { (1, 2), (2, 3), ... }*

d. Is the rule set $\subseteq A \times A$?

Properties of Binary Relations

Properties of Binary Relations
Let R be a binary relation on set A (that is, $A \to A$):

Reflexive: R is said to be reflexive if $(a, a) \in R$ for all $a \in A$. In terms of the arrow diagram, this means that **every node has a loop.**

Irreflexive: A relation R on set A is irreflexive if, for all $a \in A$, $(a, a) \notin R$). On the arrow diagram, this means **there are no loops.**

Neither Reflexive nor Irreflexive: R is neither Reflexive nor Irreflexive if there are some loops, but not all loops, or all no-loops.

Symmetric: R is called symmetric if, for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \in R$. In terms of the arrow diagram, this means that **every arrow goes in both directions.**

Antisymmetric: R is called antisymmetric if, for all $a, b \in A$, if $a \neq b$ and $(a, b) \in R$, then $(b, a) \notin R$. In terms of the arrow diagram, this means that **arrows only go in one direction.**

Neither Symmetric nor Antisymmetric: R is neither Symmetric nor Antisymmetric if some arrows go both ways, and some do not.

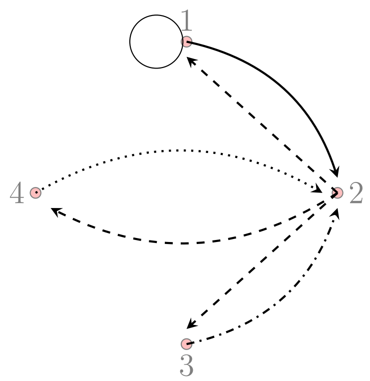
Transitive: R is transitive if, whenever $(a, b) \in R$ and $(b, c) \in R$, it must also be the case that $(a, c) \in R$. In terms of the arrow diagram, this means that **whenever you can follow two arrows to get from node a to node c, you can also get there along a single arrow.**

Not transitive: R is not transitive if it is not transitive.

Question 7

Identify the properties for each of the following relations, using only the graphs.

a.

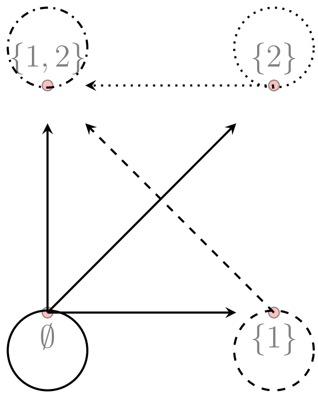


○ Reflexive ○ Irreflexive ○ Neither

○ Symmetric ○ Antisymmetric ○ Neither

○ Transitive ○ Not transitive

b.

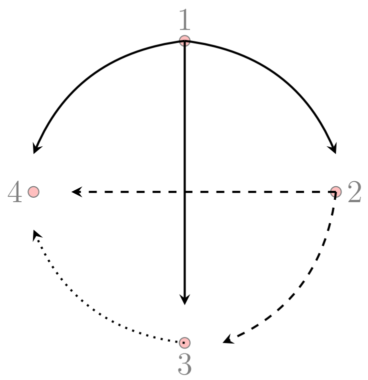


○ Reflexive ○ Irreflexive ○ Neither

○ Symmetric ○ Antisymmetric ○ Neither

○ Transitive ○ Not transitive

c.

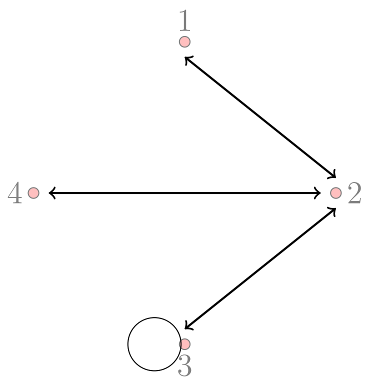


○ Reflexive ○ Irreflexive ○ Neither

○ Symmetric ○ Antisymmetric ○ Neither

○ Transitive ○ Not transitive

d.



○ Reflexive ○ Irreflexive ○ Neither

○ Symmetric ○ Antisymmetric ○ Neither

○ Transitive ○ Not transitive

Question 8
Identify the properties for each of the following relations, using the rules.

a. $R = \{(a,b) \in \{1,2,3,4\} \times \{1,2,3,4\} :$
$\{(1,1),(1,2),(2,1),(2,3),(2,4),(3,2),(4,2)\}\}$

○ Reflexive ○ Irreflexive ○ Neither
○ Symmetric ○ Antisymmetric ○ Neither
○ Transitive ○ Not transitive

b. $R = \{(a,b) \in \wp(\{1,2\}) \times \wp(\{1,2\}) : \{$
$(\emptyset,\emptyset),\quad (\emptyset,\{1\}),\quad (\emptyset,\{2\}),\quad (\emptyset,\{1,2\})$
$(\{1\},\{1\}),\quad (\{1\},\{1,2\}),$
$(\{2\},\{2\}),\quad (\{2\},\{1,2\}),$
$(\{1,2\},\{1,2\})$
$\}\}$

○ Reflexive ○ Irreflexive ○ Neither
○ Symmetric ○ Antisymmetric ○ Neither
○ Transitive ○ Not transitive

c. $R = \{(a,b) \in \{1,2,3,4\} \times \{1,2,3,4\} : \{$
$(1,2),(1,3),(1,4)$
$(2,3),(2,4)$
$(3,4)$
$\}\}$

○ Reflexive ○ Irreflexive ○ Neither
○ Symmetric ○ Antisymmetric ○ Neither
○ Transitive ○ Not transitive

d. $R = \{(a,b) \in \{1,2,3,4\} \times \{1,2,3,4\} :$
$\{(1,2),(2,1),(2,3),(2,4),(3,2),(3,3),(4,2)\}\}$

○ Reflexive ○ Irreflexive ○ Neither
○ Symmetric ○ Antisymmetric ○ Neither
○ Transitive ○ Not transitive