Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own "dropbox" on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

3.1: Functions and Relations: Functions and Function Properties

1. Given the set $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$ and the set... $\{(1, a), (2, a), (3, c), (4, b)\}$... [1]

   a. What is the domain?

   b. What is the range?

   c. Draw the arrow diagram.

   d. Is this one-to-one?

   e. Is this onto?

   f. Is this a function?

   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.

2. Given the set $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$ and the set... $\{(1, c), (2, a), (3, b), (4, c), (2, d)\}$... **??**

   a. What is the domain?

   b. What is the range?

   c. Draw the arrow diagram.

   d. Is this one-to-one?

   e. Is this onto?

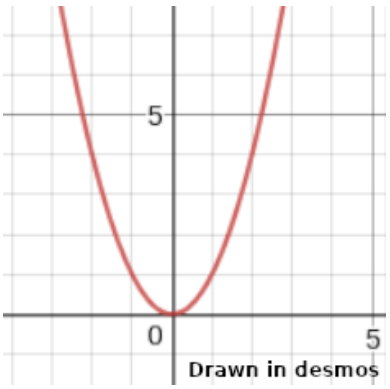
   f. Is this a function?

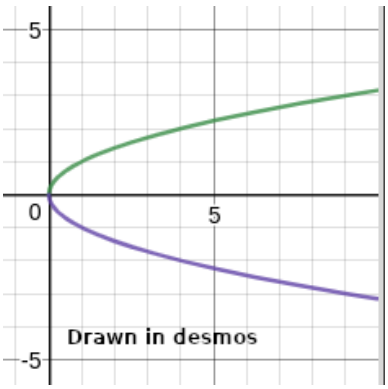
   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.

[1] From Discrete Mathematics, 7th edition, Johnsonbaugh, pg 132

3. Given the set $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$ and the set... $\{(1, c), (2, d), (3, a), (4, b)\}$... ??
   a. What is the domain?
   b. What is the range?
   c. Draw the arrow diagram.
   d. Is this one-to-one?
   e. Is this onto?
   f. Is this a function?
   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.
4. Given the set $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$ and the set... $\{(1, b), (2, b), (3, b), (4, b)\}$... ??
   a. What is the domain?
   b. What is the range?
   c. Draw the arrow diagram.
   d. Is this one-to-one?
   e. Is this onto?
   f. Is this a function?
   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.

5. Given this function and its inverse, answer the following questions:





Function
$y = x^2$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$

Inverse
$y = \pm\sqrt{x}$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$

a. Is the function one-to-one?
b. Is the function onto?
c. Is the function invertible? (That is, is the inverse a function?)

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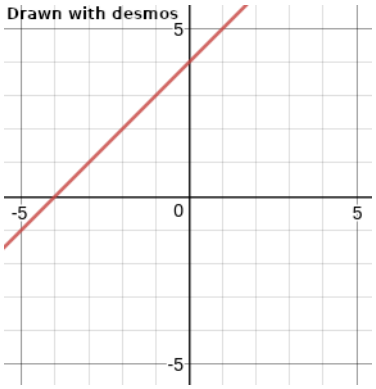
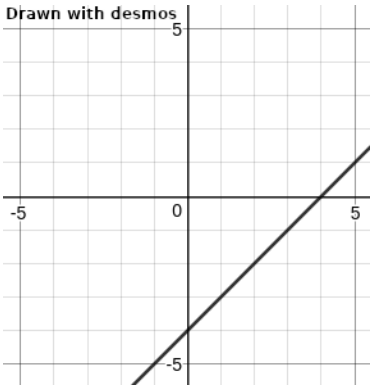
6. Given this function and its inverse, answer the following questions:





Function
$y = x + 4$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$

Inverse
$y = x - 4$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$

a. Is the function one-to-one?
b. Is the function onto?
c. Is the function invertible? (That is, is the inverse a function?)

Functions and Relations: Functions and Function Properties - Answer key

Hints

- A function is onto if every element from the codomain is an output of something from the domain.
- A function is one-to-one if no elements in the codomain is the output of two or more inputs from the domain.
- If a function is both onto and one-to-one, then it is invertible.
- An invertible function means that its inverse is also a function.

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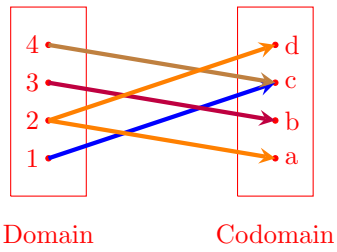
1. $\{(1, a), (2, a), (3, c), (4, b)\}$
   a. What is the domain? Domain: $\{a, b, c\}$
   b. What is the range? Range: $\{1, 2, 3, 4\}$
   c. Draw the arrow diagram.



   d. Is this one-to-one? No: The element "c" is the output of 1 and 4.
   e. Is this onto? Yes: Every element of the codomain is an output of something.
   f. Is this a function? This is not a function, as 2 leads to two different outputs.
   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain. n/a

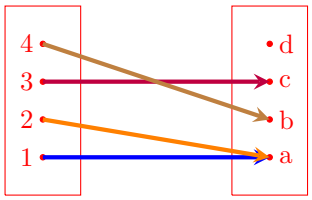
By Rachel Singh, last updated October 1, 2018

2. $\{(1, c), (2, a), (3, b), (4, c), (2, d)\}$

   a. What is the domain? Domain: $\{1, 2, 3, 4\}$

   b. What is the range? Range: $\{a, b, c, d\}$

   c. Draw the arrow diagram.



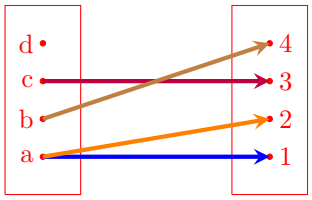
Domain Codomain

   d. Is this one-to-one? No. "a" is the output of 1 and 2.

   e. Is this onto? No. "d" is not the output of anything.

   f. Is this a function? Yes, everything from the domain maps to the codomain, and no item from the domain points to more than 1 item. However, this function is not invertible (the inverse is NOT a function.)

   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.



Domain Codomain

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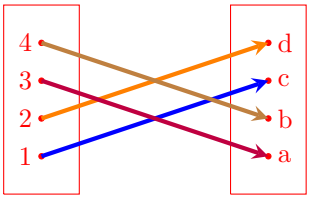
3. $\{(1, c), (2, d), (3, a), (4, b)\}$

   a. What is the domain? Domain: $\{1, 2, 3, 4\}$

   b. What is the range? Range: $\{a, b, c, d\}$

   c. Draw the arrow diagram.



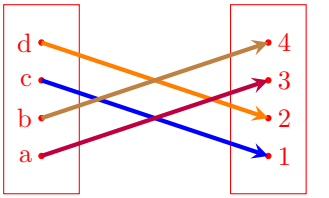
Domain Codomain

   d. Is this one-to-one? Yes.

   e. Is this onto? Yes.

   f. Is this a function? Yes, and it is invertible.

   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain.



Domain Codomain

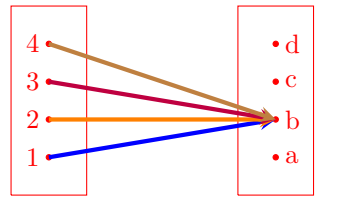
By Rachel Singh, last updated October 1, 2018

4. $\{(1, b), (2, b), (3, b), (4, b)\}$

   a. What is the domain? Domain: $\{1, 2, 3, 4\}$

   b. What is the range? Range: $\{b\}$

   c. Draw the arrow diagram.



   d. Is this one-to-one? No.

   e. Is this onto? No.

   f. Is this a function? No.

   g. If it is a function, draw the diagram for its inverse. Make sure to label the domain and codomain. n/a

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5. a. Is the function one-to-one? No, each element of the codomain can be retrieved with more than one element from the domain. For example, $f(2)$ and $f(-2)$ both give us the output 4.

   b. Is the function onto? No, not every element of the codomain ($\mathbb{R}$) is an output of the function. For example, the function never outputs numbers less than 0.

   c. Is the function invertible? (That is, is the inverse a function?)
   No, the function must be both one-to-one and onto to be an invertible function. It is not invertible. This means that the inverse is not a function.

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6. a. Is the function one-to-one? Yes.

   b. Is the function onto? Yes.

   c. Is the function invertible? (That is, is the inverse a function?) Yes.