Name: _____

Chapter 7 Exam

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Grading Scheme

#	Question	Weight	Points Received
1	Basic terms	15%	
2	Paths	15%	
3	Cycles	10%	
4	Eulerian Circuit	15%	
5	Isomorphism (2)	10%	
6	Minimal Spanning Tree	20%	
7	Planarity	5%	
8	Adjacency Matrix	10%	
9	Extra Credit Questions	+4%	

By Rachel Singh, last updated September 14, 2018

Cheatsheet

Terminology:

- **Walk:** Vertices may repeat. Edges may repeat (Closed or Open)
- **Trail:** Vertices may repeat. Edges cannot repeat (Open)
- **Circuit:** Vertices may repeat. Edges cannot repeat (Closed)
- **Path:** Vertices cannot repeat. Edges cannot repeat (Open)
- **Cycle:** Vertices cannot repeat. Edges cannot repeat (Closed) [a]

- **Euler path:** is a path that uses every edge of a graph exactly once.
- **Euler circuit:** is a circuit that uses every edge of a graph exactly once. [b]

- Simple graphs G and H are **isomorphic** if there is a one-to-one and onto function f from the nodes of G to the nodes of H such that $\{v, w\}$ is an edge of G if and only if $\{f(v), f(w)\}$ is an edge of H.
- A simple, connected graph is called **planar** if there is a way to draw it on a plane so that no edges cross.

[a] https://math.stackexchange.com/questions/655589/what-is-difference-between-cycle-path-and-circuit-in-graph-theory

[b] http://jlmartin.faculty.ku.edu/ jlmartin/courses/math105-F11/Lectures/chapter5-part2.pdf

Spanning Tree Algorithm [a]

- Begin with a simple, connected graph G_0.
- For each $i \geq 1$, as long as there is a cycle in G_{i-1}, choose an edge e in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting e from G_{i-1}.
- The final result G_k will be a spanning tree of G_0. This is a spanning tree.

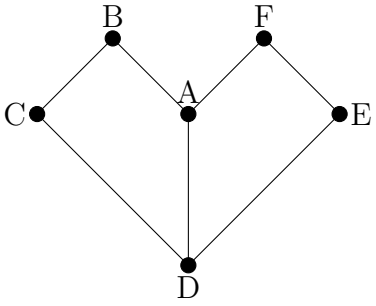
[a]Discrete Mathematicsl, Ensley and Crawley

Prim's Minimal Spanning Tree Algorithm [a]

- Given a connected, simple graph G with $n+1$ nodes...
- Let v_0 be any node in G, and let $T_0 = \{v_0\}$ be a tree with one node and no edges.
- For each k from $\{1, 2, ..., n\}$...
  - Let $E_k = \{e$ an edge in $G : e$ has one endpoint in T_{k-1} and the other endpoint not in T_{k-1}.
  - e_k be the edge in E_k with the smallest weight. (In case of a tie, choose any edge of the smallest weight.)
  - Let T_k be the tree obtained by adding edge e_k (along with its node not already in T_{k-1}) to T_{k-1}.
- T_n is the tree returned by the algorithm.

[a]Discrete Mathematicsl, Ensley and Crawley

Question 1: Basic Terms □ 0 □ 1 □ 2 □ 3 □ 4



a. Identify all vertices (nodes):

b. Identify all edges:

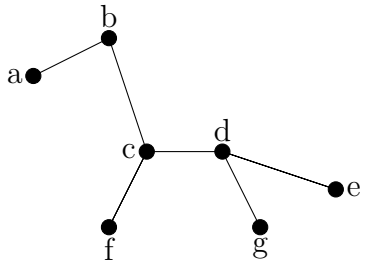
c. Identify the degrees:

$deg(A) =$ $deg(B) =$ $deg(C) =$

$deg(D) =$ $deg(E) =$ $deg(F) =$

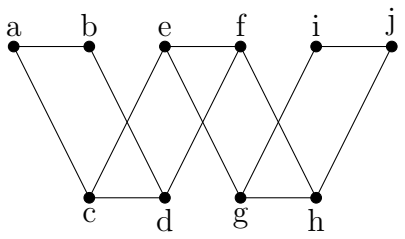
Question 2: Paths □ 0 □ 1 □ 2 □ 3 □ 4

Given this graph, find a path of length 4 and write it down.



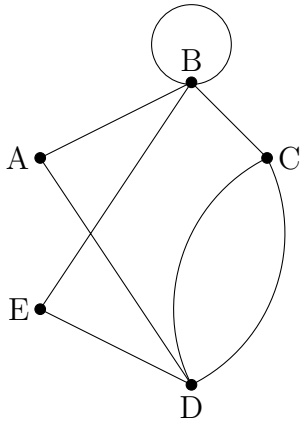
Question 3: Cycles □ 0 □ 1 □ 2 □ 3 □ 4

Given this graph, find a cycle of length 6 and write it down.



Question 4: Eulerian Circuit □ 0 □ 1 □ 2 □ 3 □ 4

Given this graph, find a Eulerian Circuit. [1]

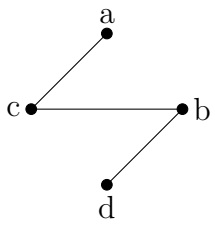


[1] http://jlmartin.faculty.ku.edu/ jlmartin/courses/math105-F11/Lectures/chapter5-part2.pdf

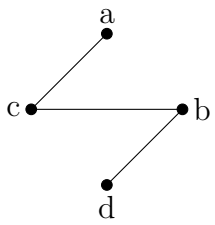
Question 5: Isomorphism □ 0 □ 1 □ 2 □ 3 □ 4

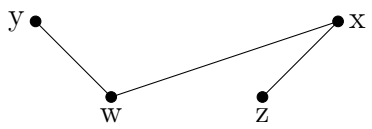
Determine whether each pair of graphs is isomorphic. If they are isomorphic, write out the mapping between nodes from $v_i \mapsto w_j$. If it is not isomorphic, write "not isomorphic".

a.



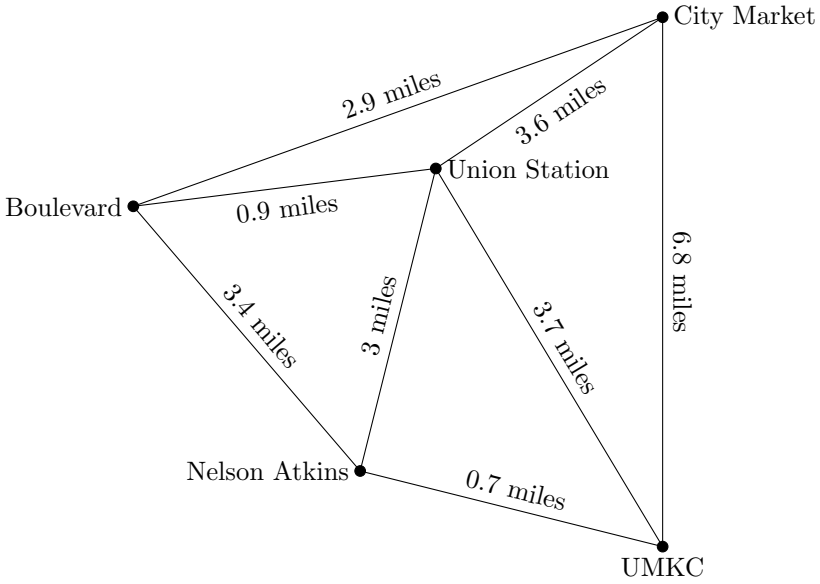
b.





Question 6: Minimal Spanning Tree □ 0 □ 1 □ 2 □ 3 □ 4

Using **Prim's Minimal Spanning Tree Algorithm** and starting at node *"UMKC"*, find a minimal spanning tree for the following graph.

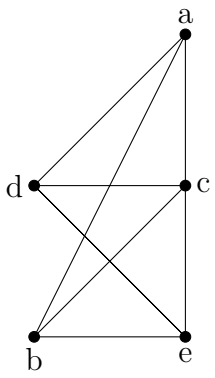


By Rachel Singh, last updated September 14, 2018

Extra page for problem 6

Question 7: Planarity □ 0 □ 1 □ 2 □ 3 □ 4

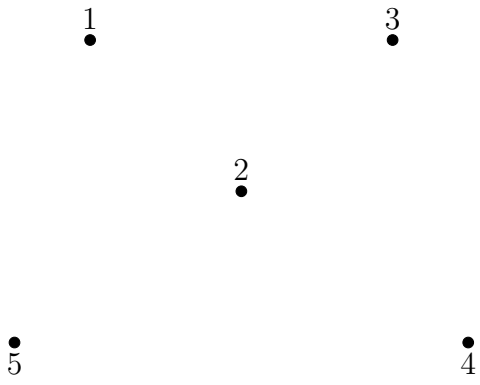
Redraw the following graph to be planar. Make sure to label your vertices!



Question 8: Adjacency Matrix □ 0 □ 1 □ 2 □ 3 □ 4

Draw the directed graph for the given adjacency matrix.

Rows \ Columns	1	2	3	4	5
1	0	1	0	0	1
2	1	0	0	1	1
3	1	0	1	1	0
4	0	0	0	0	1
5	0	0	0	0	0



Question 9: Extra Credit □ 0 □ 1 □ 2 □ 3 □ 4

a. **CS 210, Chapter 1:** Given an implication $p \to q$, what does the negation, $\neg(p \to q)$, simplify to? (Hint: The negation is not also an implication.)

b. **CS 210, Chapter 2:** An integer n is odd if it can be written in the form...:

c. **CS 210, Chapter 3:** Write out the power set of A, $\wp(A)$, if $A = \{x, y\}$

d. **CS 210, Chapter 4:** Draw the inverse of the following function.