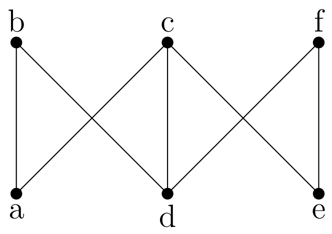
Exam 3 Preview CS 211 Fall 2017

Chapter 7.1: Graph Theory

abc% Question 1: Basic Terms □ 0 □ 1 □ 2 □ 3 □ 4



a. Identify all vertices (nodes):

b. Identify all edges:

c. Identify the degrees:
   $deg(a) =$ $deg(b) =$
   $deg(c) =$ $deg(d) =$
   $deg(e) =$
   $deg(f) =$

d. What is the maximum degree?

abc% Question 2: Terminology ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Match the terms and the definitions.

a: ____ b: ____ c: ____ d: ____ e: ____

f: ____ g: ____ h: ____

Terms:

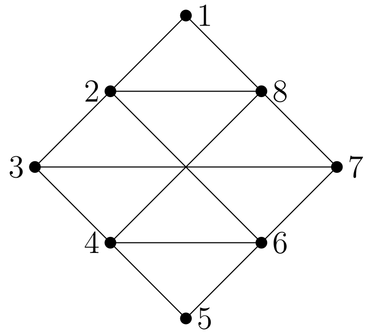
a. Loop
b. Parallel edge
c. Closed walk
d. Path
e. Trail
f. Circuit
g. Connected graph
h. Cycle

Definitions:

1. A closed trail
2. A walk where the beginning and ending nodes are the same
3. A walk with no repeated edges
4. A graph where there is a walk between any pair distinct nodes.
5. Two edges that have the same two endpoints
6. A walk with no repeated vertices
7. An edge that begins and ends at the same node
8. A nontrivial circuit where the only repeated node is the first/last one.

abc% Question 3: Cycles □ 0 □ 1 □ 2 □ 3 □ 4

For the following graph:



Find a cycle of length 4.

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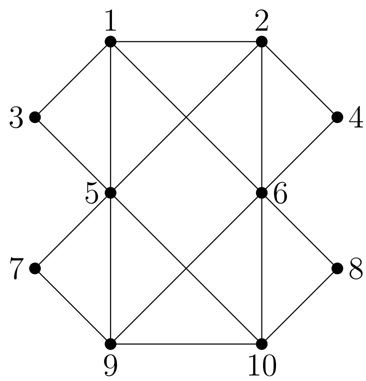
abc% Question 4: Connected, Eulerian □ 0 □ 1 □ 2 □ 3 □ 4

Identify a Eulerian [1] trail or circuit for the following graph:



[1] A Eulerian trail/circuit is a trail or circuit is one where every edge is traversed

Chapter 7.2: Proofs About Graphs and Trees

Spanning Tree Algorithm

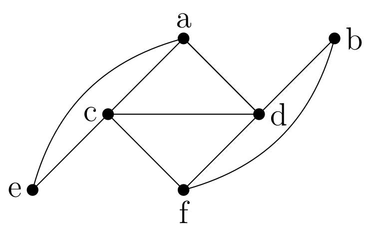
- Begin with a simple, connected graph G_0.
- For each $i \geq 1$, as long as there is a cycle in G_{i-1}, choose an edge e in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting e from G_{i-1}.
- The final result G_k will be a spanning tree of G_0. This is a spanning tree.

Prim's Minimal Spanning Tree Algorithm

- Given a connected, simple graph G with $n + 1$ nodes...
- Let v_0 be any node in G, and let $T_0 = \{v_0\}$ be a tree with one node and no edges.
- For each k from $\{1, 2, ..., n\}$...
  - Let $E_k = \{e$ an edge in G : e has one endpoint in T_{k-1} and the other endpoint not in T_{k-1}.
  - e_k be the edge in E_k with the smallest weight. (In case of a tie, choose any edge of the smallest weight.)
  - Let T_k be the tree obtained by adding edge e_k (along with its node not already in T_{k-1}) to T_{k-1}.
- T_n is the tree returned by the algorithm.

abc% Question 5: Spanning trees ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Build out a spanning tree for the following graph.



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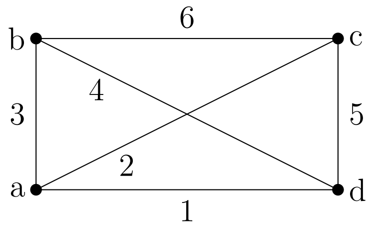
abc% Question 6: Minimal spanning trees ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Starting at vertex a, find a minimal spanning tree for the following graph and specify the total weight.



Chapter 7.3: Isomorphism and Planarity

abc% Question 7: Isomorphism (7.3) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Determine if the following graph pairs are isomorphic. [2]

a.

[2]Simple graphs G and H are isomorphic if there is a one-to-one and onto function f from the nodes of G to the nodes of H such that $\{v, w\}$ is an edge of G if and only if $\{f(v), f(w)\}$ is an edge of H.

b.

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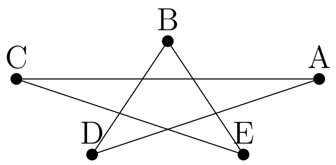
abc% Question 8: Planarity (7.3) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

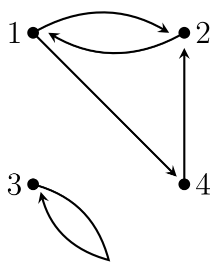
Redraw the following graph to be Planar, [3] make sure to label the vertices.

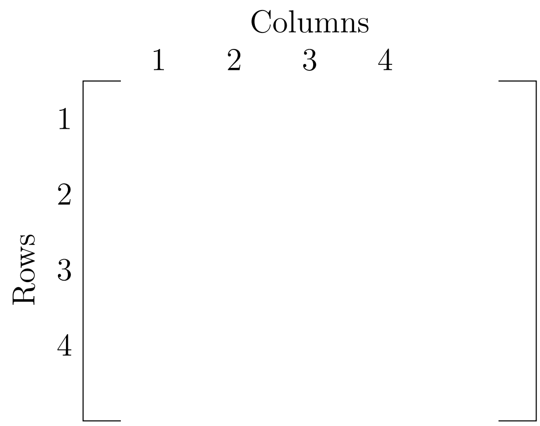


Chapter 7.4: Connections to Matrices and Relations

abc% Question 9: Adjacency matrix (7.4) ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

Build the adjacency matrix for the following directed graph.





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[3] A simple, connected graph is called planar if there is a way to draw it on a plane so that no edges cross

Key

Question 1

a. Vertices: a, b, c, d, e, f

b. Edges: [a,b], [a,c], [b,d], [c,d], [c,e], [d,f], [e,f]

c.
$deg(a) = 2$ $deg(b) = 2$
$deg(c) = 3$ $deg(d) = 3$
$deg(e) = 2$
$deg(f) = 2$

d. Maximum degree: 3

Question 2

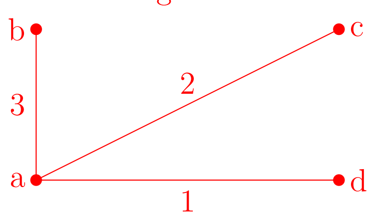
a: 7	b: 5	c: 2	d: 6	e: 3
f: 1	g: 4	h: 8		

Question 3
Many solutions; example: 3, 2, 8, 4, 3

Question 4
1, 2, 4, 6, 8, 10, 9, 7, 5, 3, 1, 5, 2, 6, 10, 5, 9, 6, 1

Question 5
Many solutions

Question 6
Using Prim's algorithm, there is only one, with edges [a, d], [a, c], [a, b] and a total weight of 6.

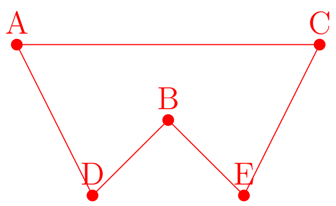


Question 7

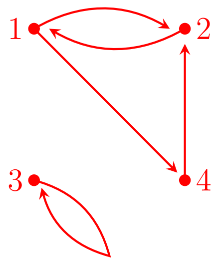
a. Isomorphic

b. Not Isomorphic

Question 8



Question 9



Columns

Rows	1	2	3	4
1	0	1	0	1
2	1	0	0	0
3	0	0	1	0
4	0	1	0	0