

Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group.**

Names:

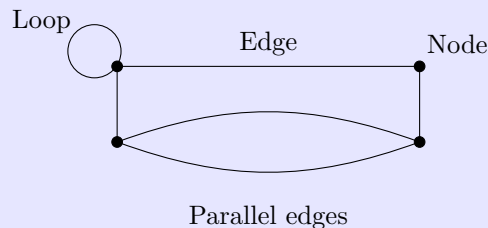
7.6 Graph Theory

7.6.1 Terminology

Since we're introducing a new concept, Graph Theory, we need to go over the various terms so that we can communicate about these graphs properly.

Graphs, nodes, and edges

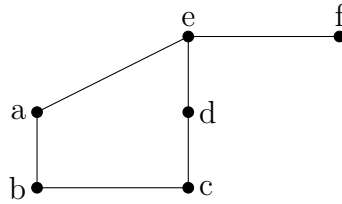
- **Graph:** A graph is a type of diagram that contains *vertices* (aka nodes) and *edges*.



- **Node:** A vertex of the graph, drawn as a dot.
 - **Adjacent nodes:** Two nodes that are connected by an edge.
 - **Node degree:** The amount of edges that are connected to a node. Loops are counted twice.
- **Edge:** A line that connects two nodes together.
 - **Parallel edges:** Two edges that have the same two endpoints.
 - **Loop:** An edge that begins and ends at the same node, creating a loop.
 - $[a, b]$ is used to indicate an edge with a and b as endpoints, though direction can be either way.

Question 1

Identify each item for the graph G given.



- a. How many nodes (vertices) are there?

- b. How many edges are there?

- c. Write down the degree of each node:

Vertex v	$deg(v)$
a	
b	
c	
d	
e	
f	

- d. The **maximum degree** of a graph is the highest $deg(v)$ value. What is this graph's maximum degree?

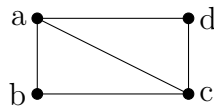
- e. The **minimum degree** of a graph is the lowest $deg(v)$ value. What is this graph's minimum degree?

Types of walks

- **Walk:** A series of alternating nodes and edges, traversing between adjacent nodes.
 - **Closed walk:** When the beginning and ending node of a walk are the same.
 - **Length of a walk:** The amount of edges in the walk.
 - **Trivial walk:** A walk of length 0.
 - **Path:** A walk with no repeated vertices.
 - **Trail:** A walk with no repeated edges.
 - * **Circuit:** A closed trail.
 - **Trivial circuit:** A circuit with one vertex and no edges.
 - **Cycle:** A nontrivial circuit where the only repeated node is the first/last one.
 - * **Eulerian:** A trail or circuit where every edge is traversed.

Question 2

Answer the following questions, using the graph H given.



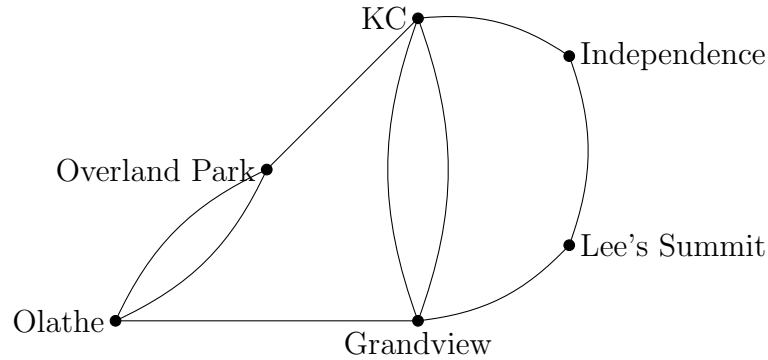
- a. Come up with several walks from a to c . Write all steps (each node visited). Also label the **length** of each walk.

- b. Come up with a **closed walk**, beginning and ending at a . You can choose to visit all nodes or not.

- c. Come up with a **path**, where no vertices are repeated.

Question 3

Answer the following questions, using the graph I given.



- Come up with a **trail**, a walk with no repeated edges.
- Come up with a **circuit**, a closed trail.
- Come up with a **cycle**, a circuit where the only repeated node is the first/last one..
- Identify: Did you come up with any **Eulerian Trails**?
If not, create one.
- Identify: Are there any **parallel edges**?

Simple vs. directed graphs

- **Simple graph:** A graph that has no loops or parallel edges.
- **Directed graph:** The edges in the graph are given a direction, which can only be traversed in that way.
 - Edges are denoted with parentheses (a, b) , showing that it goes from a to b .

Question 4

Draw a **Directed Graph** using the following list of edges:

$(1, 2), (2, 1), (3, 3), (4, 2)$

(Don't confuse these for points on an x, y plane that are interconnected, each ordered pair is its own set of information - beginning and end nodes.)

Connected, subgraphs

- A graph is **connected** if there is a walk between any pair of distinct nodes.
- A graph H is a **subgraph** of a graph G if all nodes and edges in H are also nodes and edges in G .
- A **connected component** of a graph G is a connected subgraph H of G such that no other connected subgraph of G containing H exists.

^a

^aDiscrete Mathematics, Ensley and Crawley

Question 5

Draw a graph that is **not connected**, and draw a **subgraph** of your graph.