Discrete Structures II: Graph Theory: Adjacency Matrices
Textbooks: Ensley & Crawley: Chapter 7.4

Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group**.

Names:

7.6 Graph Theory: Adjacency Matrices

7.6.1 Adjacency matrix

Adjacency matrix

Given a graph G with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E, we define the **adjacency matrix** of G as follows. The matrix M is an $n \times n$ array of natural numbers, which we imagine having rows and columns labelled as follows:

Columns $1, 2, ..., n$

Row 1 $\rightarrow$

Row 2 $\rightarrow$

Row 3 $\rightarrow$

Row ...

Row n $\rightarrow$

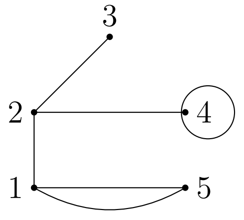
The entry in row i, column j (referred to as the (i, j) - entry of M or, more concisely, M_{ij}) is defined as

$$M_{ij} = \text{the number of edges connecting } v_i \text{ and } v_j \text{ in } G. \text{ [a]}$$

[a] Discrete Mathematics, by Ensley and Crawley

Question 1

Fill out the adjacency matrix for the following graph.



Columns

Rows	1	2	3	4	5
1					
2					
3					
4					
5					

By Rachel Singh, last updated September 30, 2018

7.6.2 Directed graphs

Directed graphs

1. A **directed graph**, like a graph, consists of a set V of vertices and a set E of edges. Each edge is associated with an ordered pair of vertices called its **endpoints**. In other words, a directed graph is the same as a graph, but the edges are described as *ordered pairs* rather than unordered pairs;

2. If the endpoints for edge e are a and b in that order, we say e is an edge **from** a **to** b, and in the diagram we draw the edge as a straight or curved arrow from a to b.

3. For a directed graph, we use (a, b) rather than $[a, b]$ to indicate an edge from a to b. This emphasizes that the edge is an **ordered pair**, by utilizing the usual notation for ordered pairs.

4. A **walk** in a directed graph in a sequence $v_1e_1v_2e_2...v_ne_nv_{n+1}$ of alternating vertices and edges that begins and ends with a vertex, and where each edge in the list between its endpoints in the proper order. (That is, e_1 is an edge from v_1 to v_2, e_2 is an edge from v_2 to v_3, and so on.) If there is no chance of confusion, we omit the edges when we describe a walk.

5. The **adjacency matrix** for a directed graph with vertices $\{v_1, v_2, ..., v_n\}$ is the $n \times n$ matrix where M_{ij} (the entry in row i, column j) is the number of edges from vertex v_i to vertex v_j.

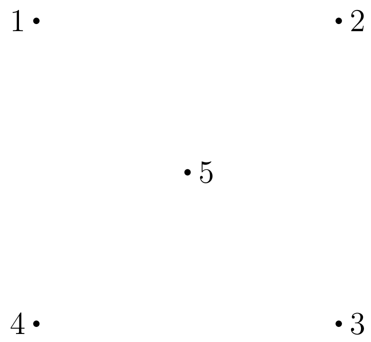
[a]

[a]Discrete Mathematics, Ensley and Crawley

Question 2

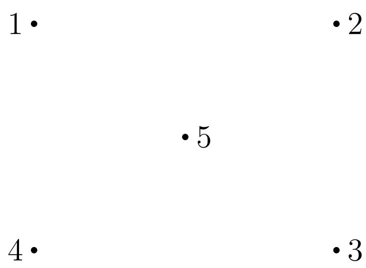
Draw a graph that corresponds to the adjacency matrix. This is a directed graph, so the matrix is not symmetric. It should be read as row $i \to$ column j. For example, row 1 shows $1 \to 2$, $1 \to 4$, and $1 \to 5$.

Rows \ Columns	1	2	3	4	5
1	0	1	0	1	1
2	0	0	0	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	0	0	1	0



Question 3

Draw a directed graph with vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{(1, 4), (1, 5), (2, 1), (3, 4), (4, 3), (5, 2)\}$.



By Rachel Singh, last updated September 30, 2018