

**Instructions:** In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group.**

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Names:

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## 7.6 Graph Theory: Graphs in Puzzles and Games

### 7.6.1 Wolves, Goats, and Cabbages

#### Rules



A traveler has three possessions: a wolf, a goat, and a cabbage. They must transport them across a river.

The catch is that, if left alone, the wolf will eat the goat, or the goat will eat the cabbage. The boat can only hold the traveller and one possession at a time. <sup>a</sup>

For this problem, we are concerned with what valid states are. We can draw a diagram to represent all possible moves between the starting point (everything at the starting location) and the ending point (everything at the ending location) to help us solve it.

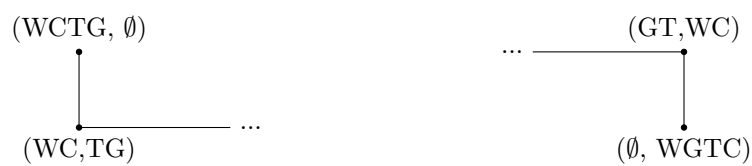
Let's use  $(WC, TG)$  to mean that the Wolf and Cabbage are left on the departing shore, and the Traveller and the Goat are on the arriving shore. If we write  $(WCTG, \emptyset)$ , then all four are on the departing shore, and nothing is on the arriving shore.

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<sup>a</sup>Discrete Mathematics, Ensley and Crawley

### Question 1

Build out a graph from the state where the Wolf, Goat, Traveller, and Cabbage are on Island 1 ( $CGTW, \emptyset$ ), to the state where all four are on Island 2 ( $\emptyset, CGTW$ ). You can use the *cabbages* visualization from <http://edu.moosader.com/visualizations/> if it helps out.



- What does  $W$  represent?
- What does  $C$  represent?
- What does  $T$  represent?
- What does  $G$  represent?
- What does  $\emptyset$  represent?
- Build out the graph to represent each state.
- List out a path:

## Question 2

Two friends have 2 gallons (8 quarts) of water in a pail. They also have two (empty) jars, one holding 5 quarts and the other 3. Using just these measuring devices, how can they split the water so that 4 quarts are in the larger jar and 4 quarts remain in the pail? <sup>1</sup>

Build out the state diagram and give the path from start  $(8, 0, 0)$  to end  $(4, 4, 0)$ .



- What does the 1st position in the vertex label represent?
- What does the 2nd position in the vertex label represent?
- What does the 3rd position in the vertex label represent?
- Draw the state diagram/graph.
- Give a Path to the solution.

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<sup>1</sup>Every node within the square also has edges pointing to two of the four corners of the square, but those have been left off.