Trees!

Written by Rachel Singh, last updated Sep 30, 2018

About

Trees are a handy structure in Data Structures, and are also a part of Graph Theory.

Topics

1. Intro to Trees
2. Propositions & Theorems
3. Spanning Tree Algorithms

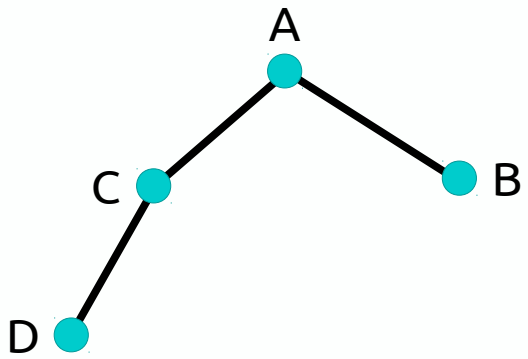
Intro to Trees

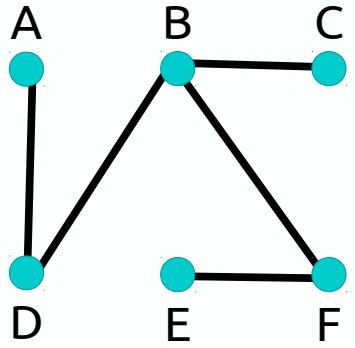
1. Intro to Trees

Connected Graph: A graph where there exists a walk between any two arbitrarily chosen nodes.

Simple Graph: A graph with no loops and no parallel edges.

Tree: A connected, simple graph with no cycles.





Notes

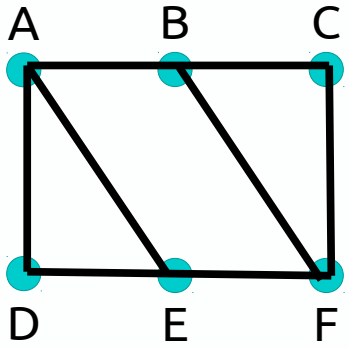
Tree: A connected, simple graph with no cycles.

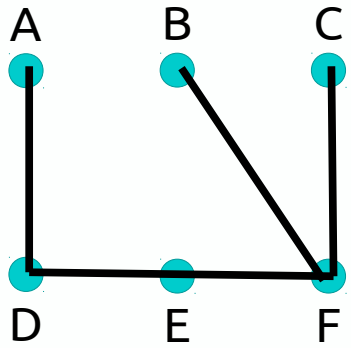
1. Intro to Trees

Connected Graph: A graph where there exists a walk between any two arbitrarily chosen nodes.

Simple Graph: A graph with no loops and no parallel edges.

Spanning Tree: Given some simple, connected graph G, a subgraph T is a spanning tree of G if T is a tree and every node in G is a node in T.





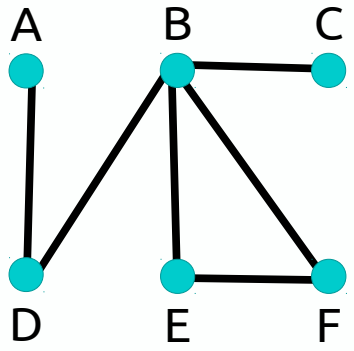
Notes

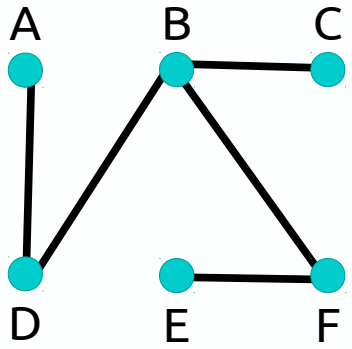
Tree: A connected, simple graph with no cycles.

Propositions & Theorems

2. Propositions & Theorems

Proposition 3: If you have a connected graph with a cycle in it, and you remove an edge from the cycle, then the graph is still connected.



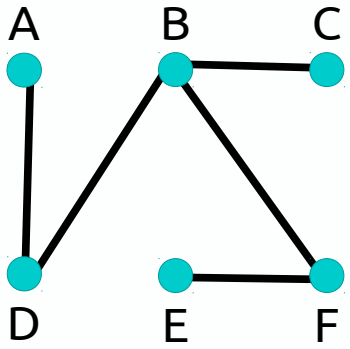


Notes

Tree: A connected, simple graph with no cycles.

2. Propositions & Theorems

Proposition 4: For a connected graph with at least one edge, if there are no cycles in the graph, then the graph has at least one vertex with degree 1.

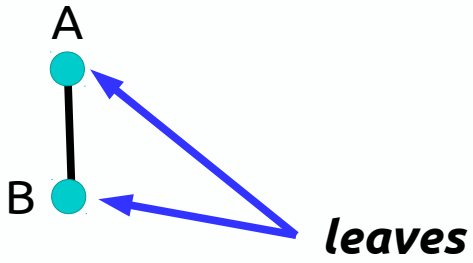


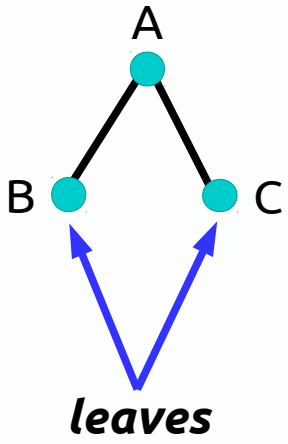
Notes

Tree: A connected, simple graph with no cycles.

2. Propositions & Theorems

Proposition 5: For any tree with at least one edge, the tree has at least two leaves.



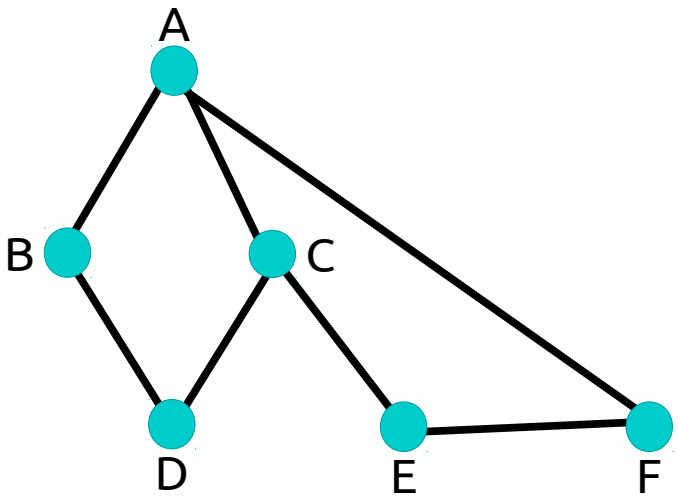


Notes

Tree: A connected, simple graph with no cycles.

2. Propositions & Theorems

Proposition 6: If a simple graph has a walk from vertex a to vertex b, then there is also a path from vertex a to vertex b.



Notes

Simple Graph: A graph with no loops and no parallel edges.

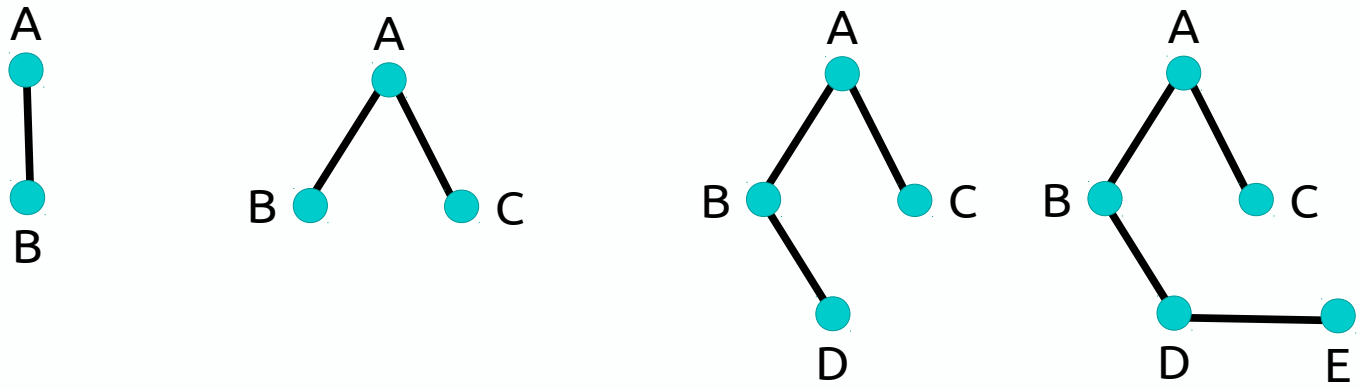
Walk: A series of alternating vertices/edges.

Trail: A walk with no repeated edges.

Path: A walk with no repeated vertices.

2. Propositions & Theorems

Theorem 7:
A tree with n edges has $n + 1$ vertices.



Notes

Simple Graph: A graph with no loops and no parallel edges.

Walk: A series of alternating vertices/edges.

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Path: A walk with no repeated vertices.

2. Propositions & Theorems

Theorem 2:
A connected graph is Eulerian if (and only if) every node has an even degree.

Notes

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Walk: A series of alternating vertices/edges.

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Path: A walk with no repeated vertices.

Spanning Tree Algorithms

3. Spanning Tree Algorithms

(Basic) Spanning Tree Algorithm:

Input: A simple, connected graph G_0.

Algorithm:

For each i >= 1, as long as there is a cycle in G_{i-1}…

Choose an edge *e* in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting *e* from G_{i-1}.

Output: The final result G_k will be a spanning tree of G_0.

From Discrete Mathematics, Ensley & Crawley, page 526

Notes

Simple Graph: A graph with no loops and no parallel edges.

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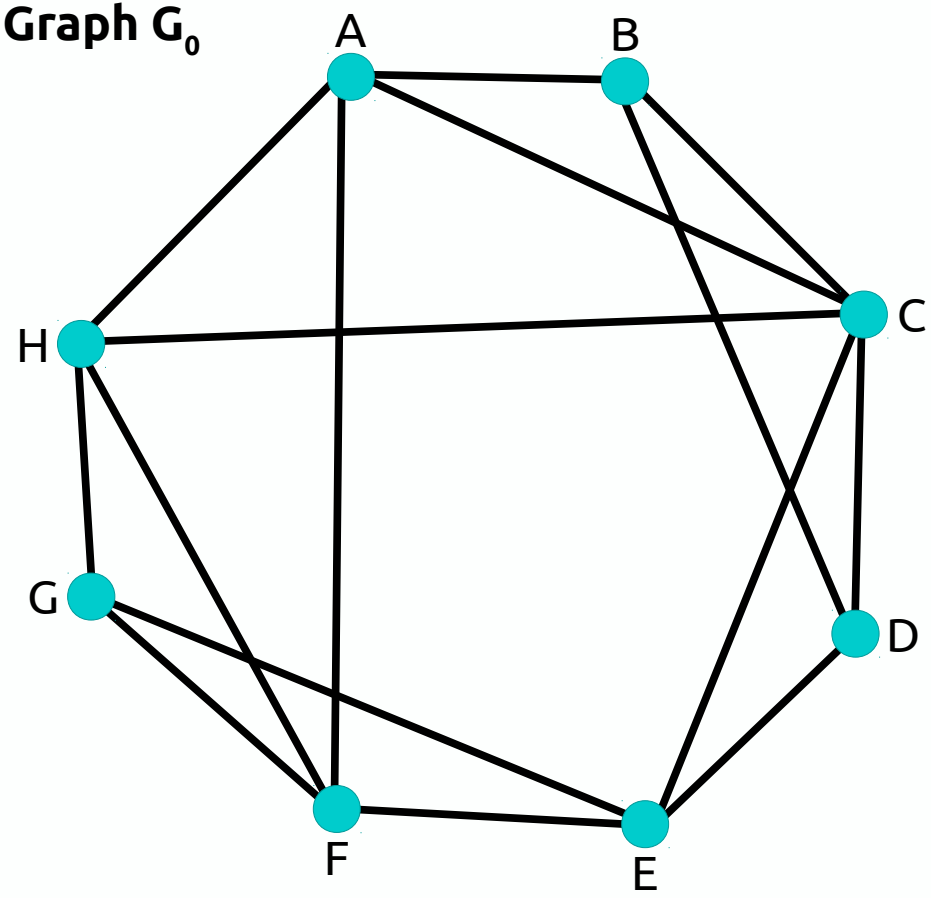
3. Spanning Tree Algorithms

Graph G_0



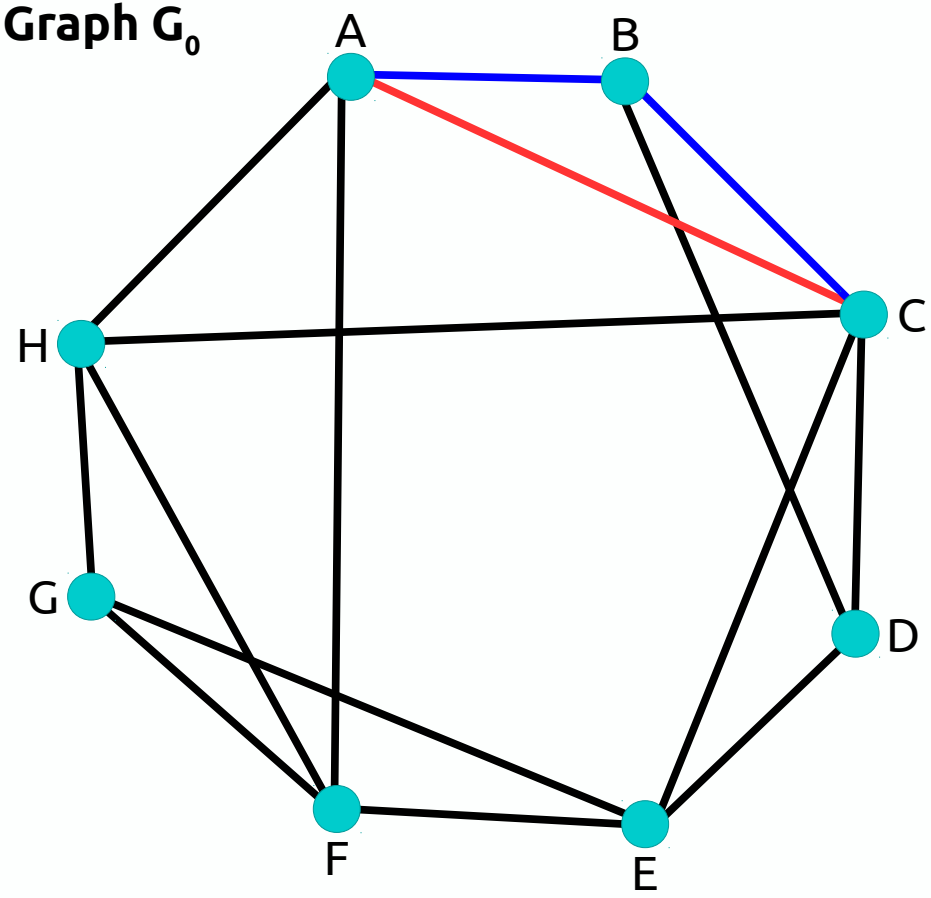
First, we begin with some simple, connected graph.

(Basic) Spanning Tree Algorithm:

- Input: A simple, connected graph G_0.
- For each i >= 1, as long as there is a cycle in G_{i-1}…
  - Choose an edge *e* in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting *e* from G_{i-1}.
- Output: The final result G_k will be a spanning tree of G_0.

3. Spanning Tree Algorithms

Graph G_0



First, we begin with some simple, connected graph.

Cycles, exist, so we will start the for loop.

Choose any arbitrary cycle in the graph.

(Basic) Spanning Tree Algorithm:

- Input: A simple, connected graph G_0.
- For each i >= 1, as long as there is a cycle in G_{i-1}…
  - Choose an edge *e* in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting *e* from G_{i-1}.
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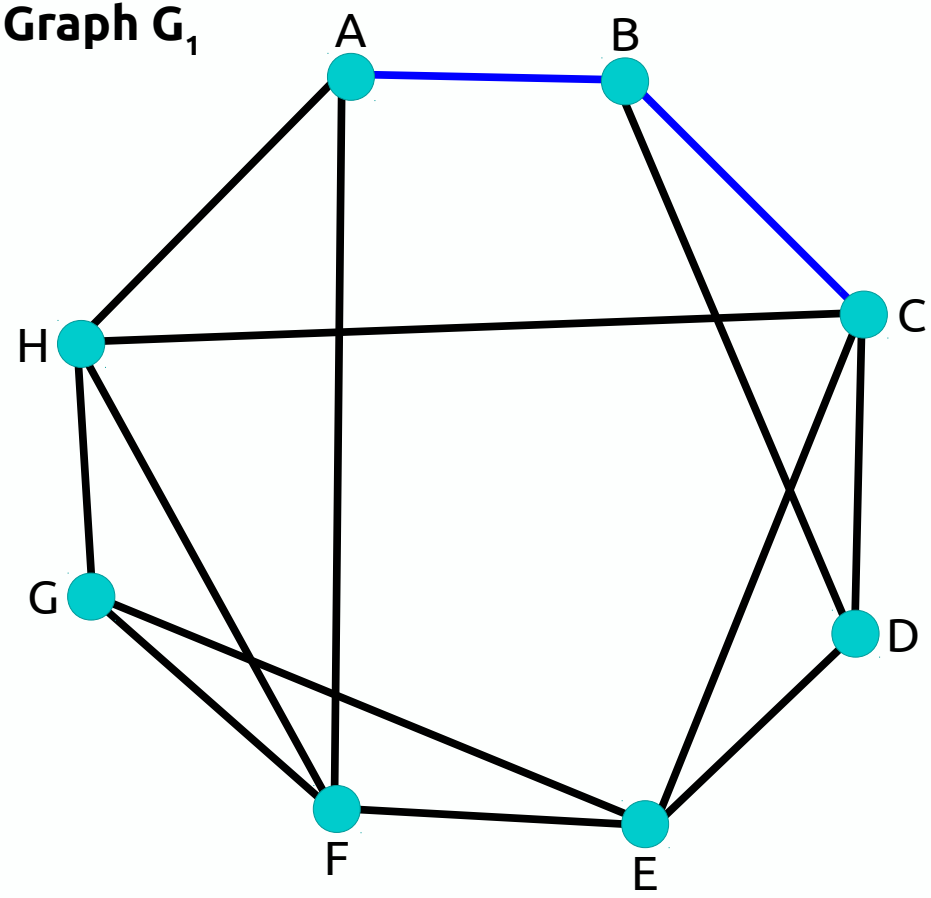
3. Spanning Tree Algorithms

Graph G_1



First, we begin with some simple, connected graph.

Choose any edge from this cycle and remove it.

This takes us from the initial state, G_0, to the next state, G_1.

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- For each i >= 1, as long as there is a cycle in G_{i-1}…
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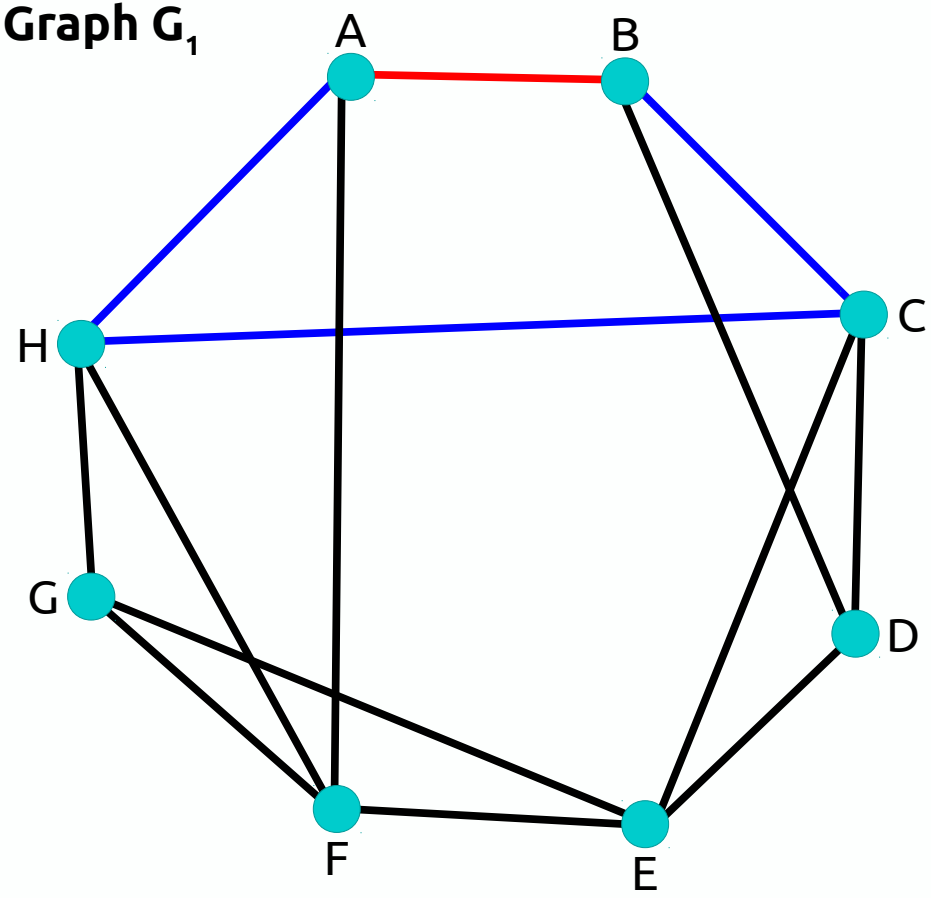
3. Spanning Tree Algorithms

Graph G_1



We still have cycles, so we continue.

Identify a cycle…

Remove an edge…

(Basic) Spanning Tree Algorithm:

- Input: A simple, connected graph G_0.
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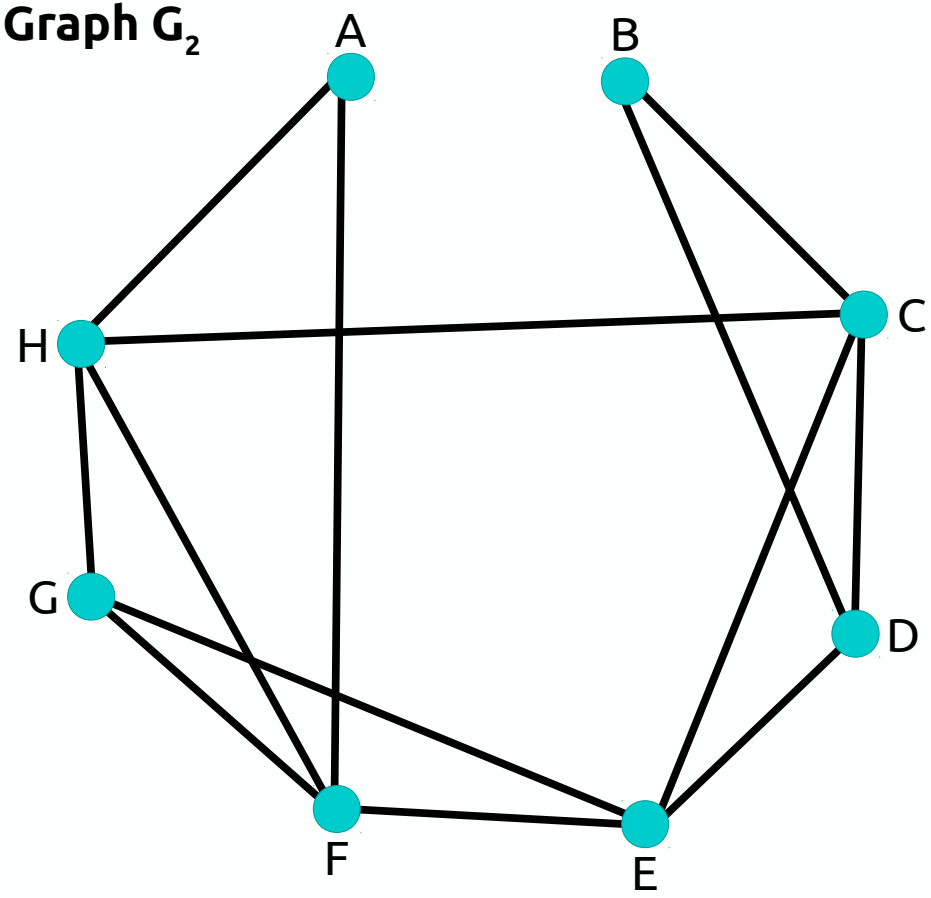
3. Spanning Tree Algorithms

Graph G_2



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Remove an edge…

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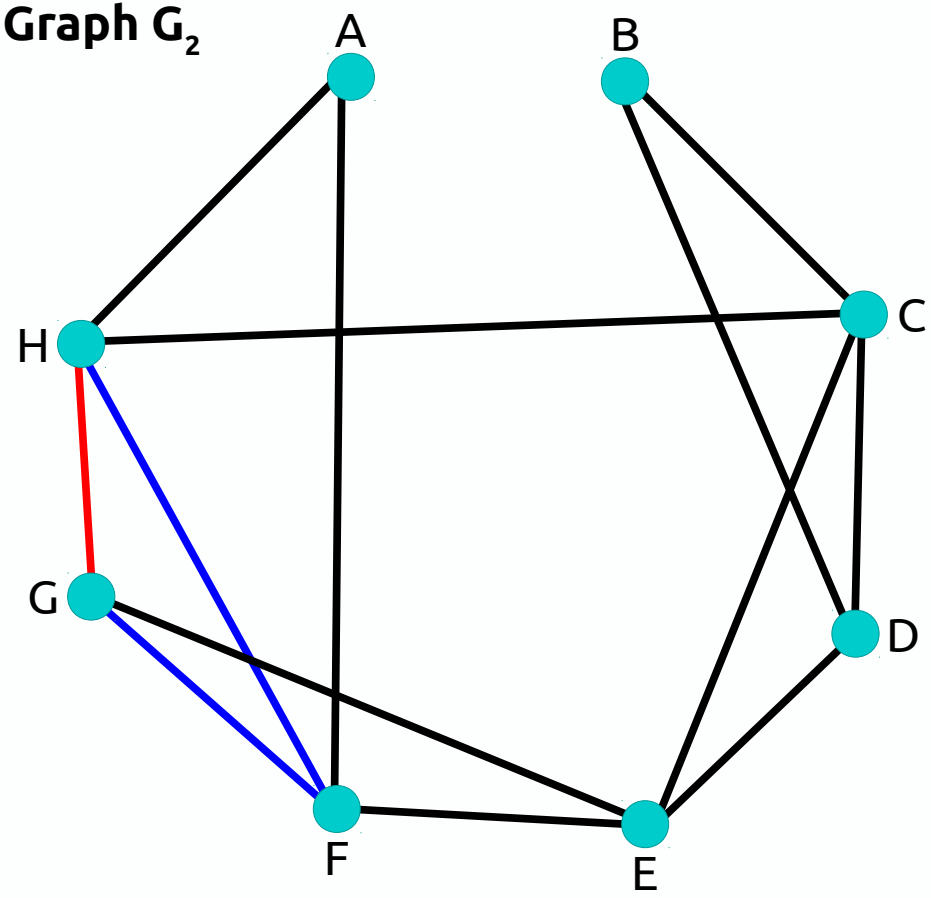
3. Spanning Tree Algorithms

Graph G_2



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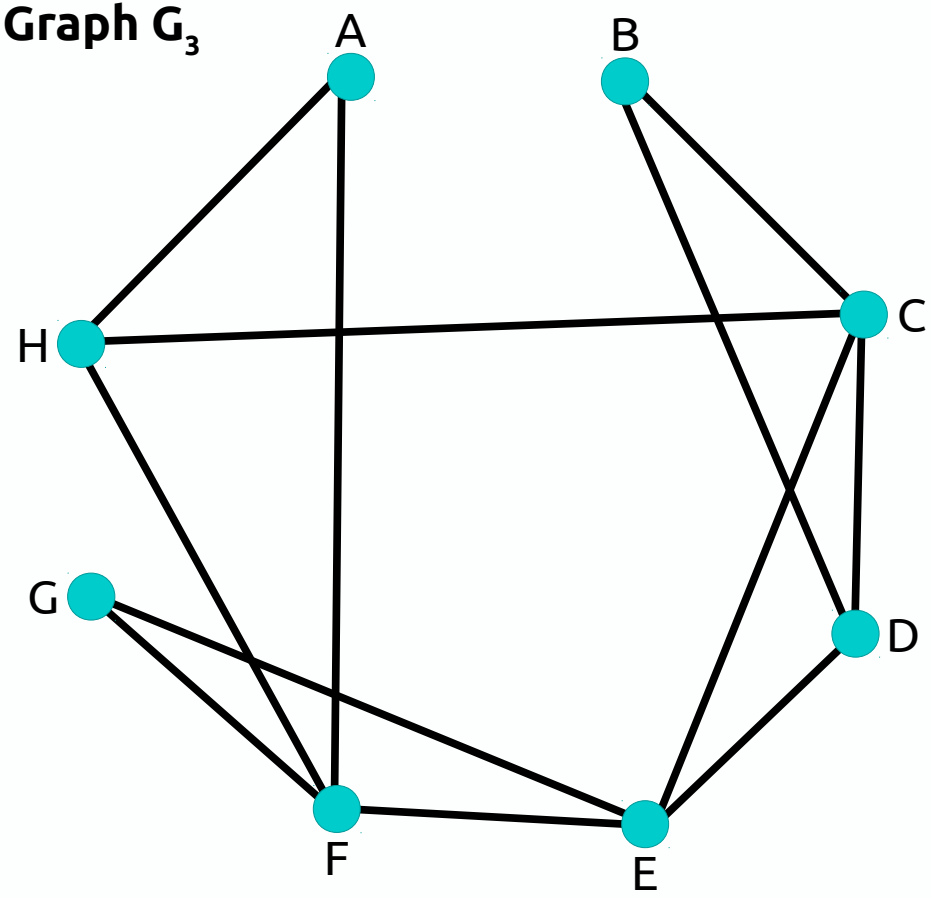
3. Spanning Tree Algorithms

Graph G_3



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Identify a cycle…

Remove an edge…

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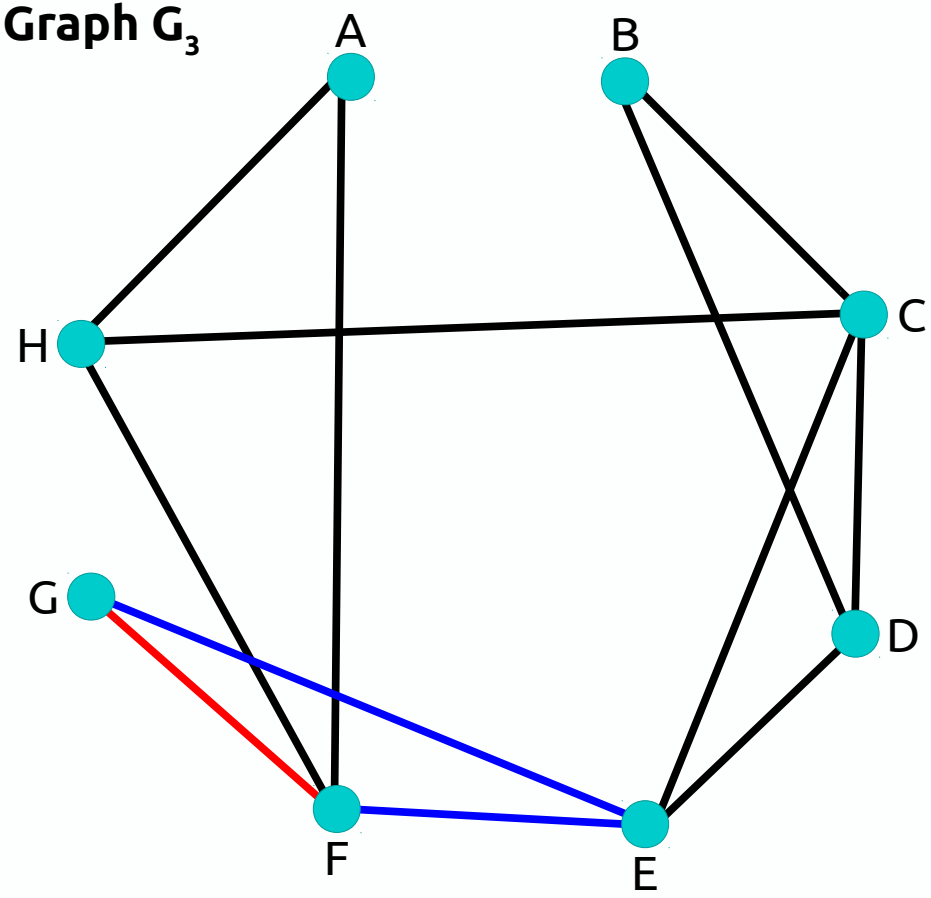
3. Spanning Tree Algorithms

Graph G_3



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Identify a cycle…

Remove an edge…

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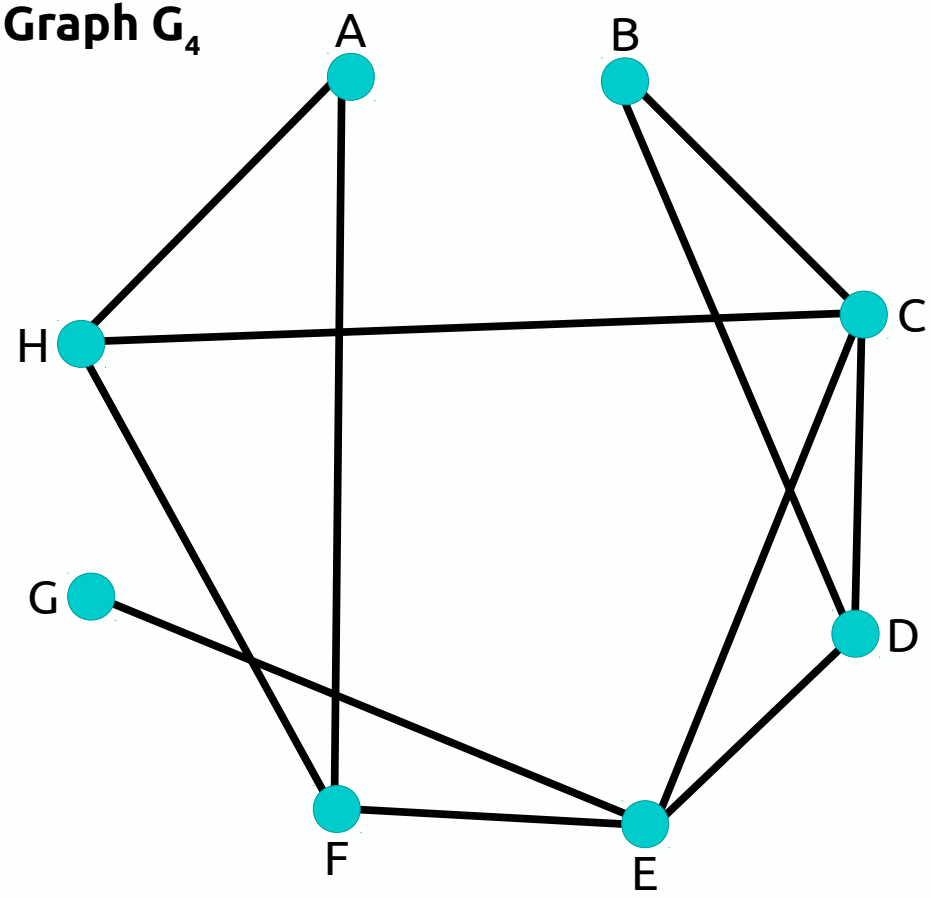
3. Spanning Tree Algorithms

Graph G_4



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Identify a cycle…

Remove an edge…

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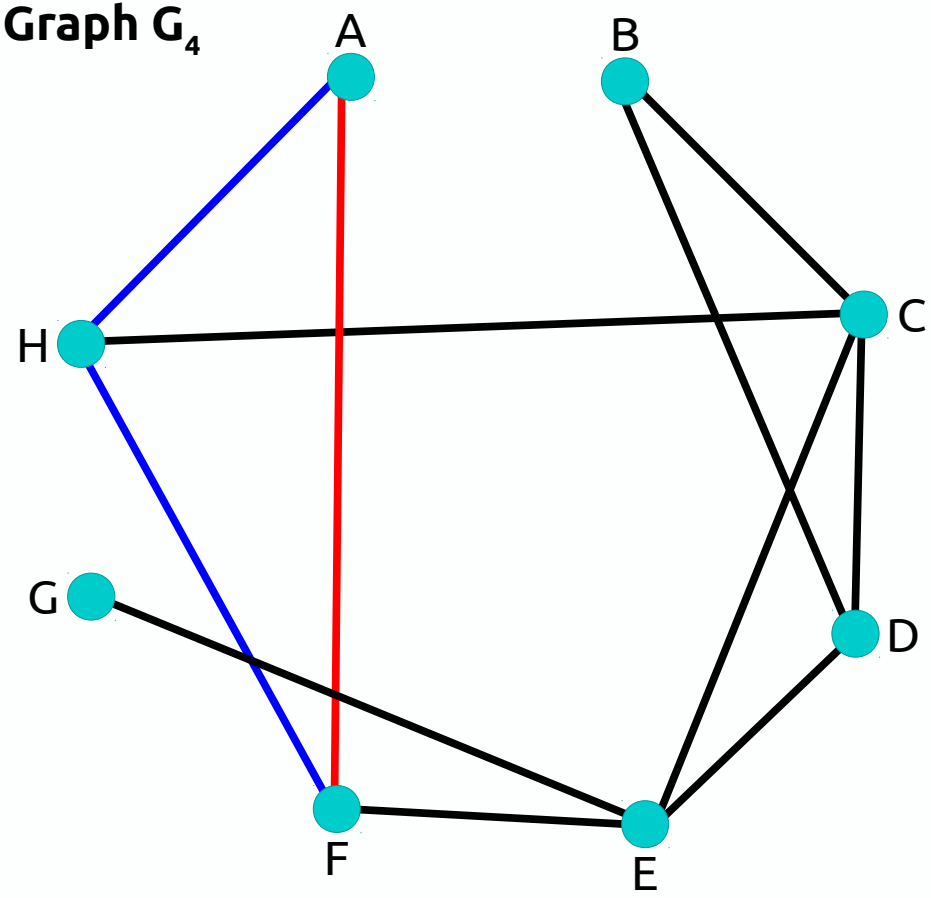
3. Spanning Tree Algorithms

Graph G_4



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Identify a cycle…

Remove an edge…

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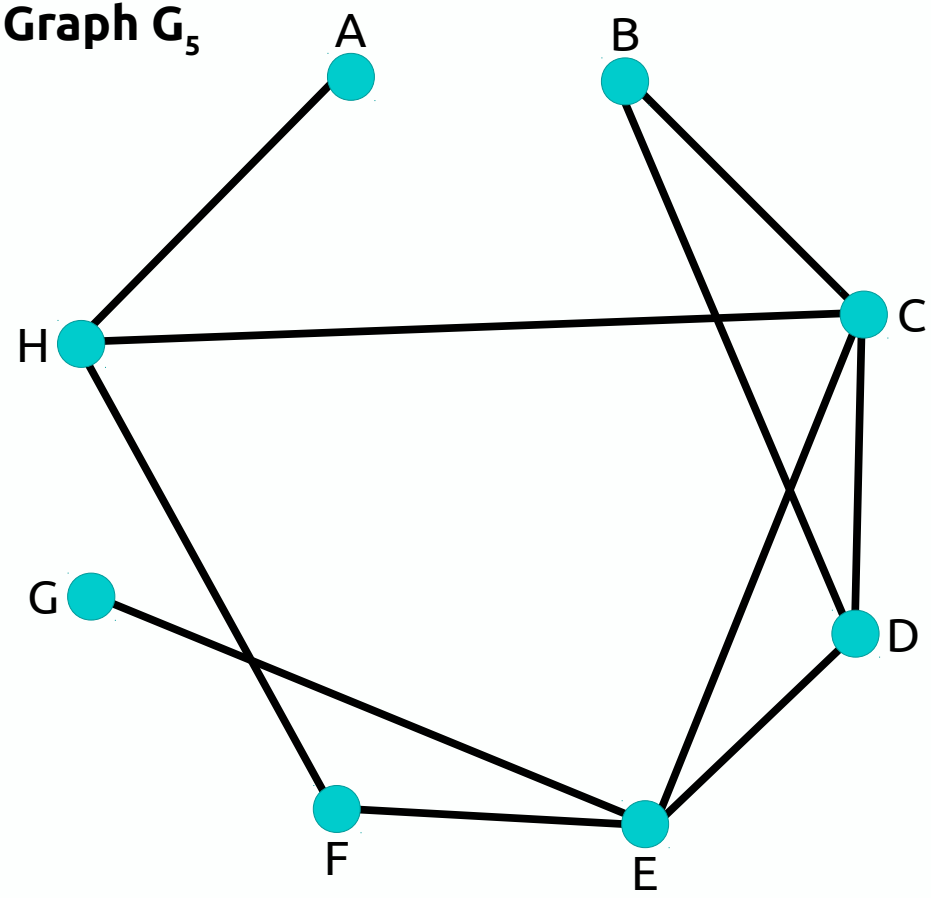
3. Spanning Tree Algorithms

Graph G_5



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Identify a cycle…

Remove an edge…

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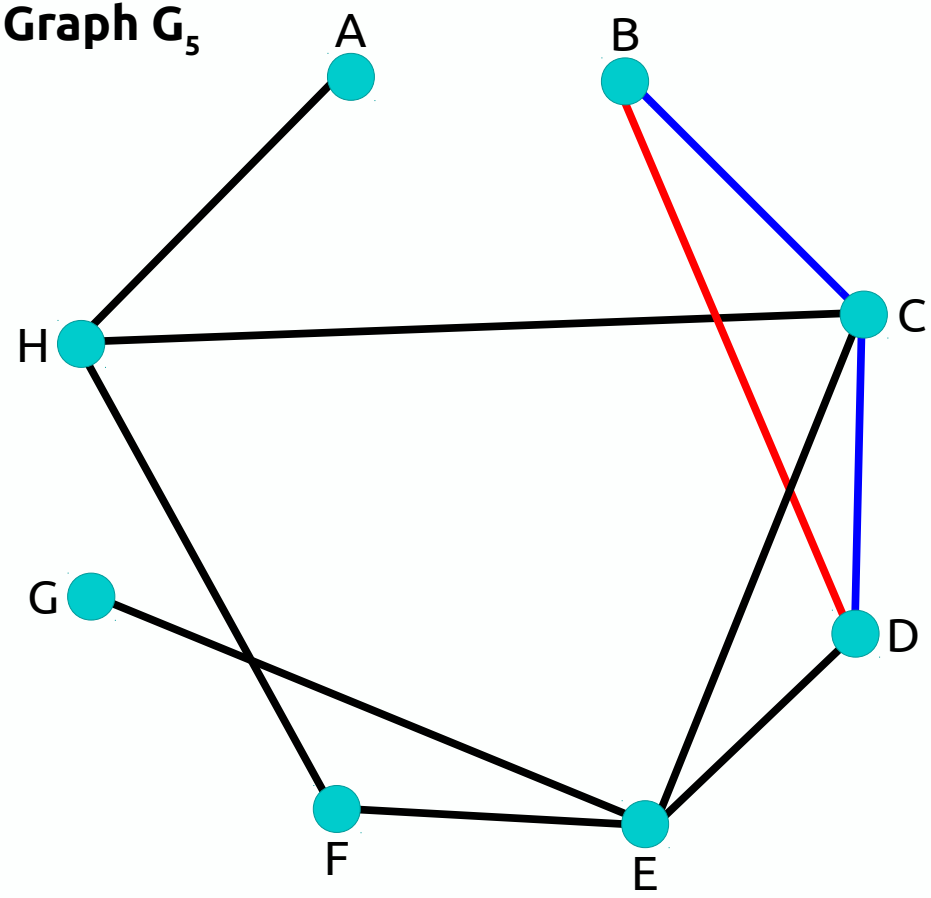
3. Spanning Tree Algorithms

Graph G_5



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Identify a cycle…

Remove an edge…

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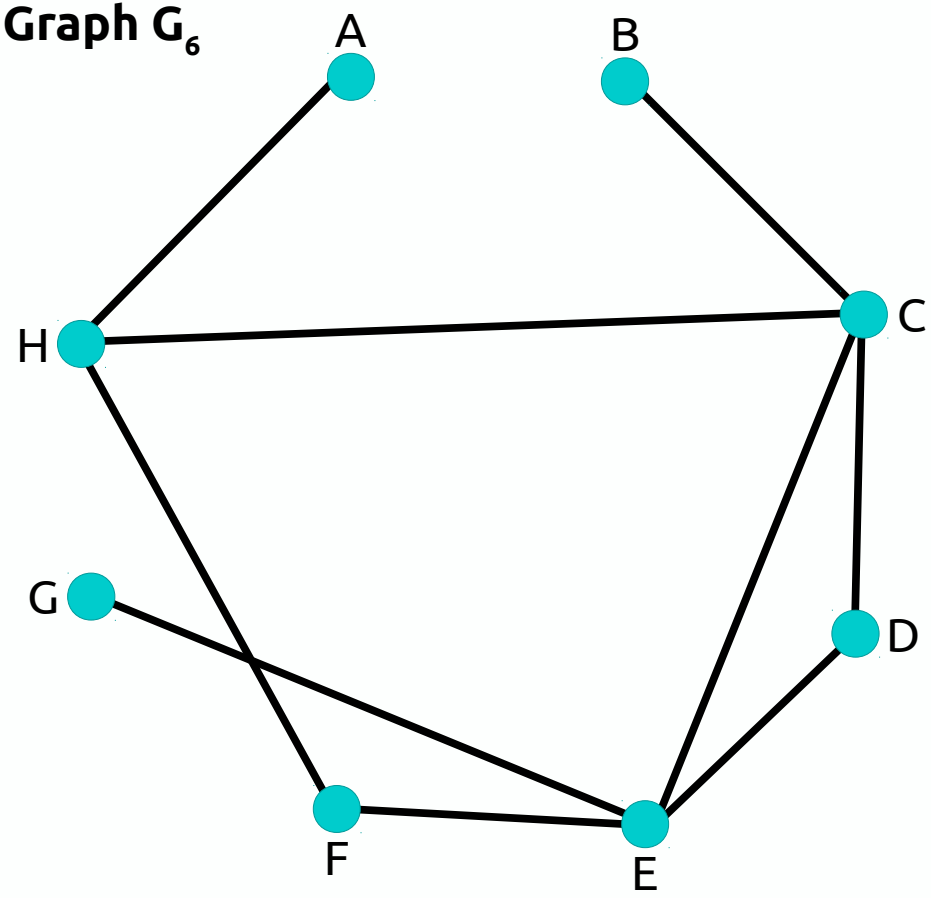
3. Spanning Tree Algorithms

Graph G_6



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Identify a cycle…

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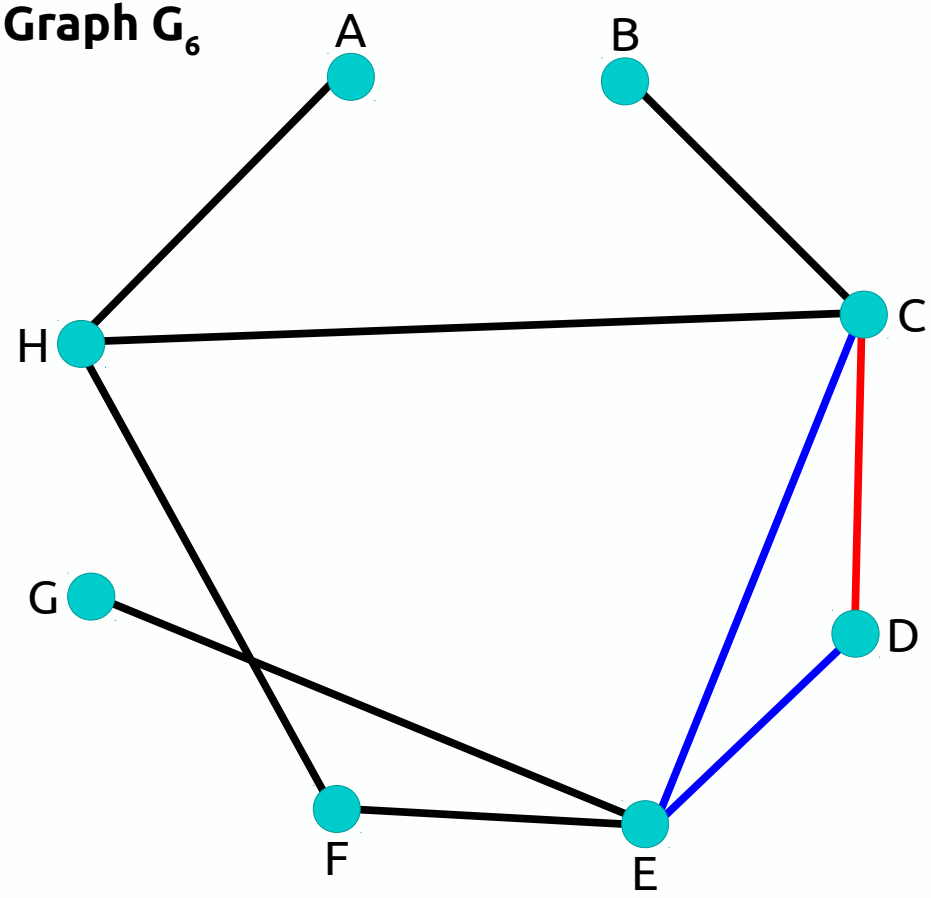
3. Spanning Tree Algorithms

Graph G_6



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Identify a cycle…

Remove an edge…

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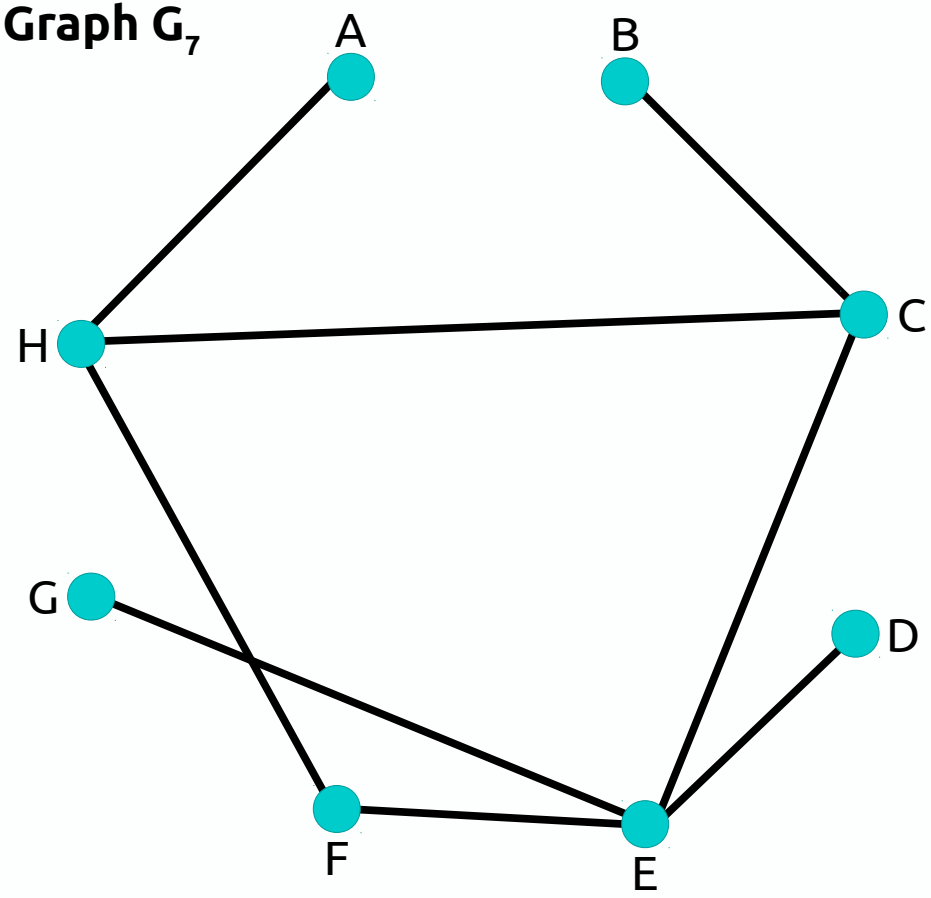
3. Spanning Tree Algorithms

Graph G_7



We still have cycles, so we continue.

Identify a cycle…

Remove an edge…

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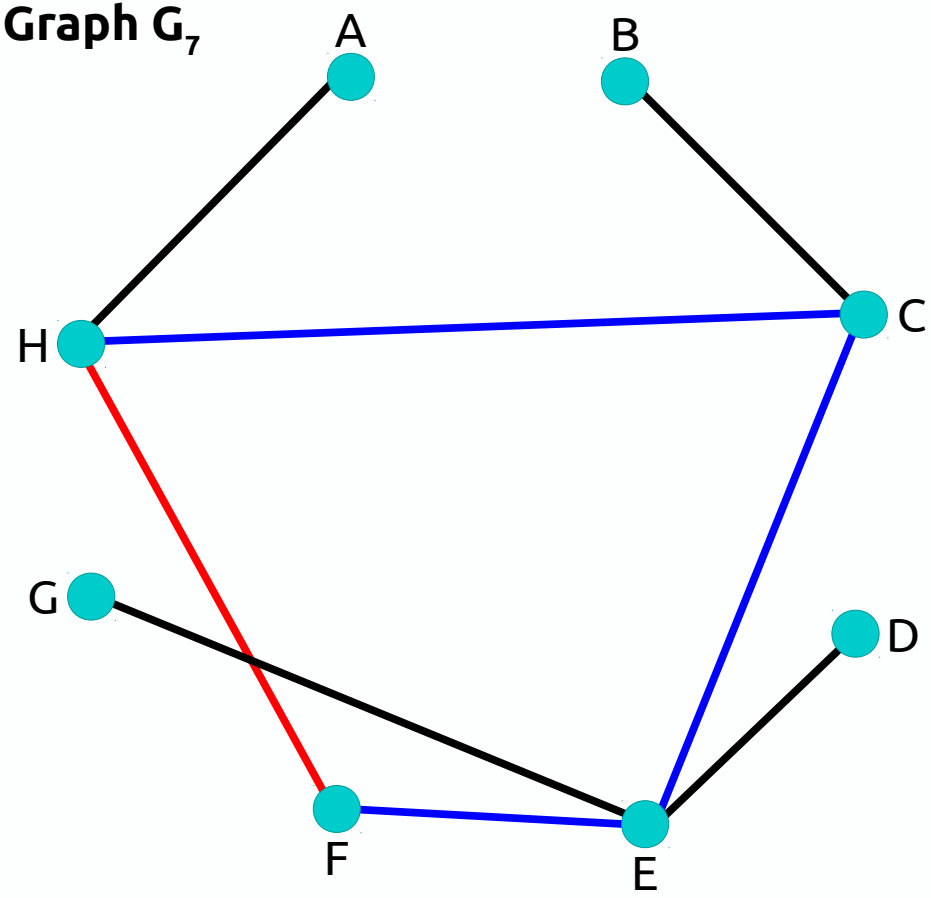
3. Spanning Tree Algorithms

Graph G_7



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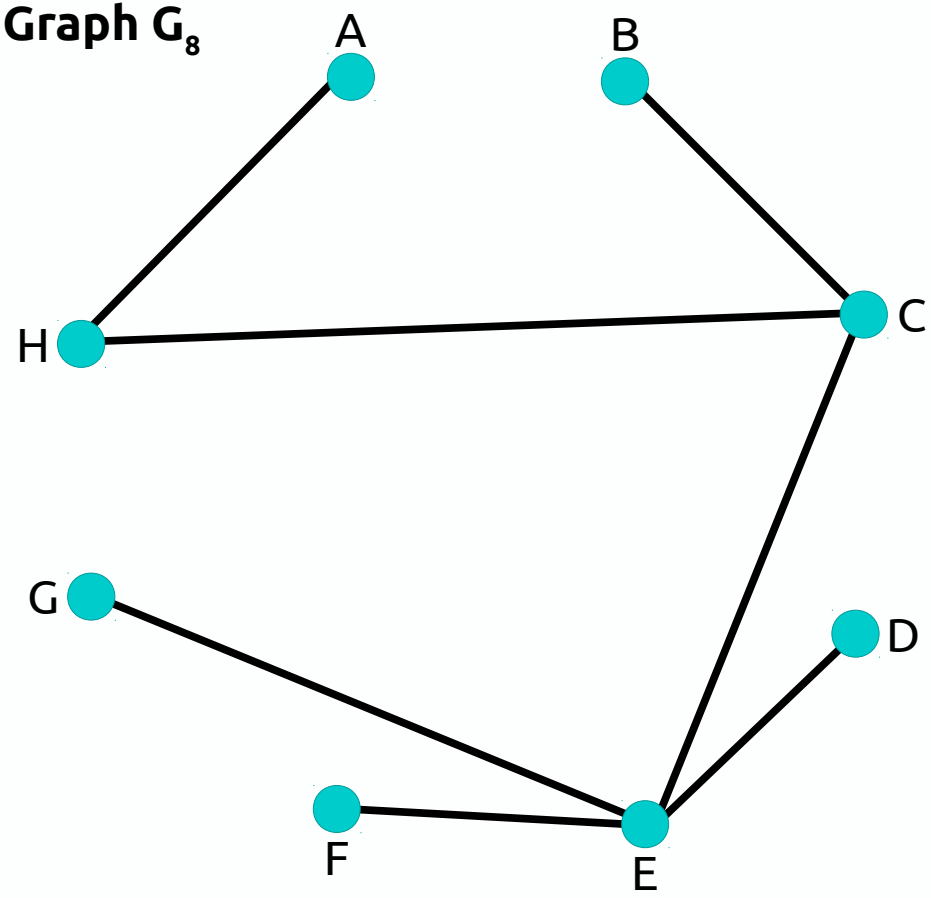
3. Spanning Tree Algorithms

Graph G_8



Now we have no more cycles, and we are done.

This is one of many possible spanning trees of the original graph, G_0.

(Basic) Spanning Tree Algorithm:

- Input: A simple, connected graph G_0.
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  - Choose an edge *e* in any cycle of G_{i-1}, and form the subgraph G_i of G_{i-1} by deleting *e* from G_{i-1}.
- Output: The final result G_k will be a spanning tree of G_0.

3. Spanning Tree Algorithms

Prim's Spanning Tree Algorithm:

Input: A simple, connected graph G with $n + 1$ nodes.

Initialize: Let v_0 be any node in G, and let $T_0 = \{v_0\}$ be a tree with one node and no edges.

For each k from {1, 2, …, n}:

- Let E_k = {e an edge in G : e has one endpoint in T_{k-1} and the other endpoint not in T_{k-1}}.
- Let e_k be the edge in E_k with the smallest weight. (In case of a tie, choose any edge of the smallest weight.)
- Let T_k be the tree obtained by adding edge e_k (along with its node not already in T_{k-1}) to T_{k-1}.

Output: The final result T_n is the tree returned by the algorithm.

From Discrete Mathematics, Ensley & Crawley, page 527

Notes

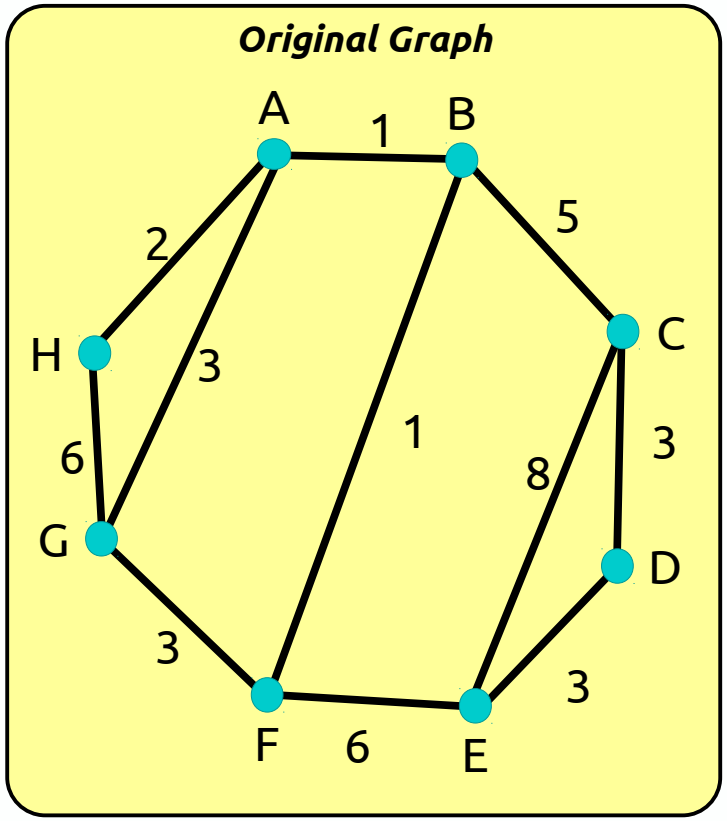
Simple Graph: A graph with no loops and no parallel edges.

Walk: A series of alternating vertices/edges.

Trail: A walk with no repeated edges.

Path: A walk with no repeated vertices.

3. Spanning Tree Algorithms



First, we begin with some simple, connected weighted graph.

We are going to build out a tree using the algorithm, and taking one edge at a time.

Notes

Prim's Spanning Tree Algorithm:

Input: A simple, connected graph G with $n + 1$ nodes.

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For each k from {1, 2, …, n}:

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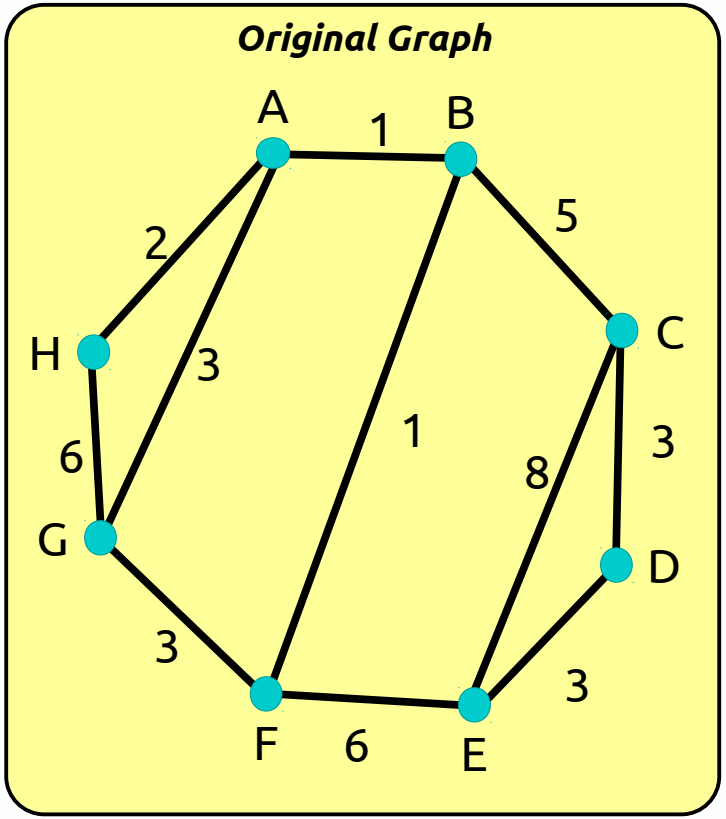
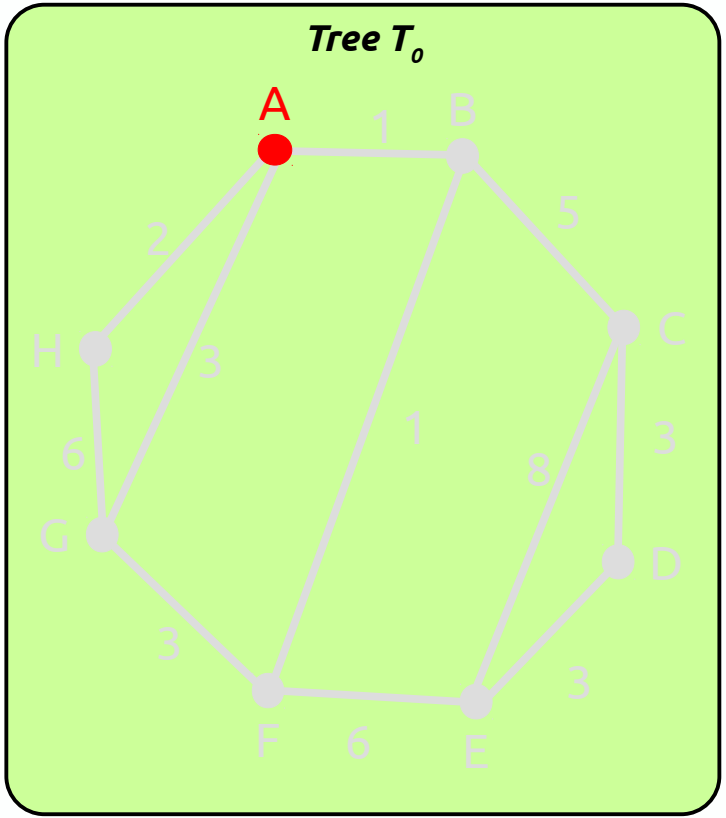
3. Spanning Tree Algorithms

We can start with any node, so I will start with node A. So $T_0 = \{v_A\}$ to start with.





Notes

Prim's Spanning Tree Algorithm:

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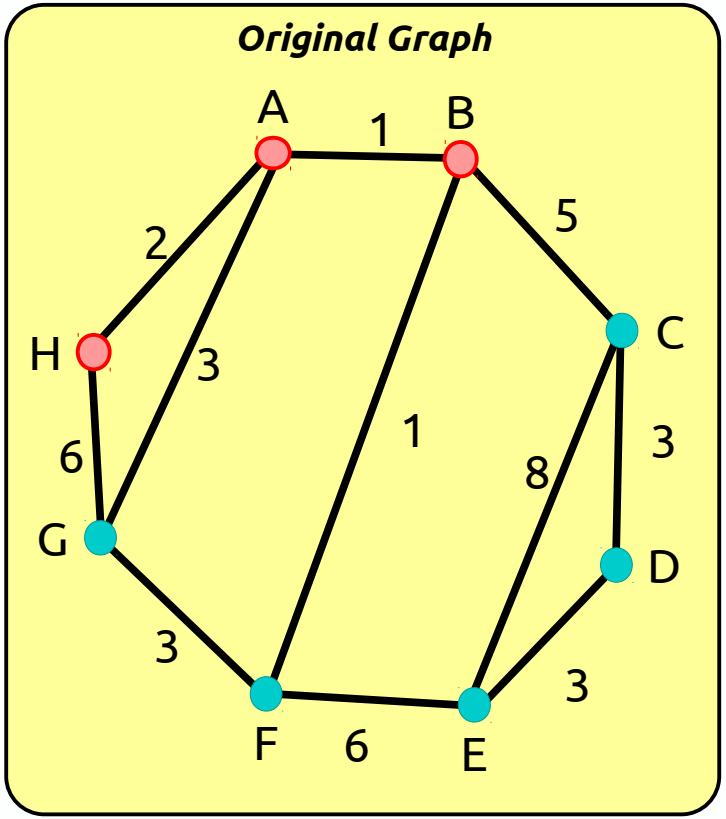
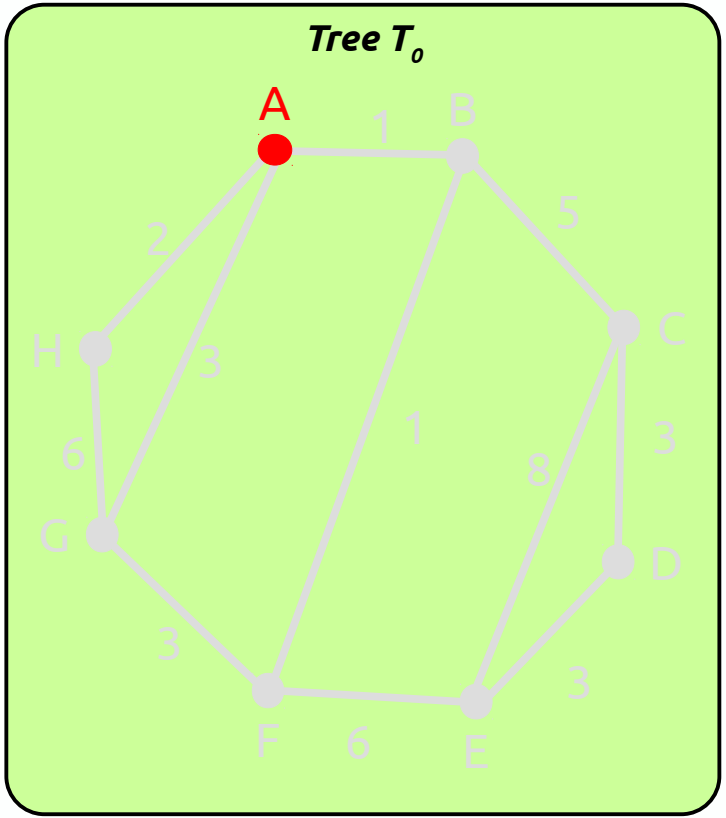
Output: The final result T_n is the tree returned by the algorithm.

3. Spanning Tree Algorithms

Next, we build E_1, the list of edges in the graph that are connected to any nodes we currently have in the tree. For now, this means just the neighbors of A.





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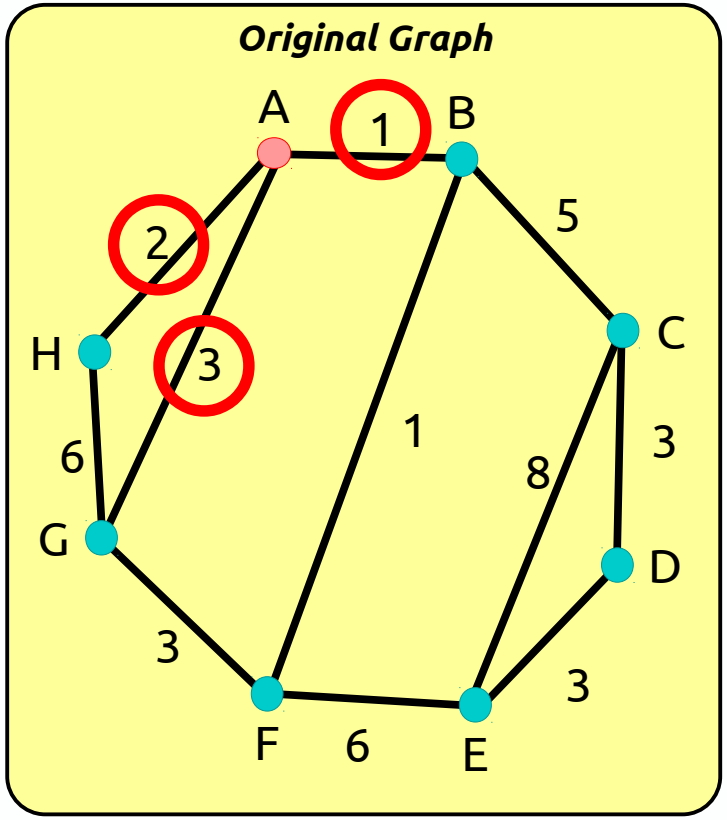
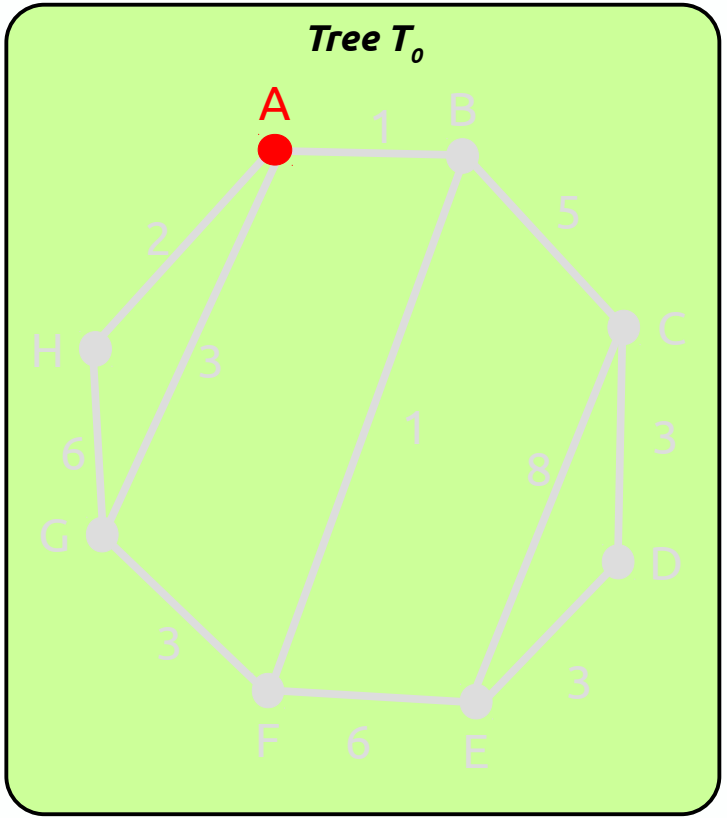
3. Spanning Tree Algorithms

The connected edges have weights 2 and 1, so we choose the smallest one and put that edge – and its endpoint B – into our tree.





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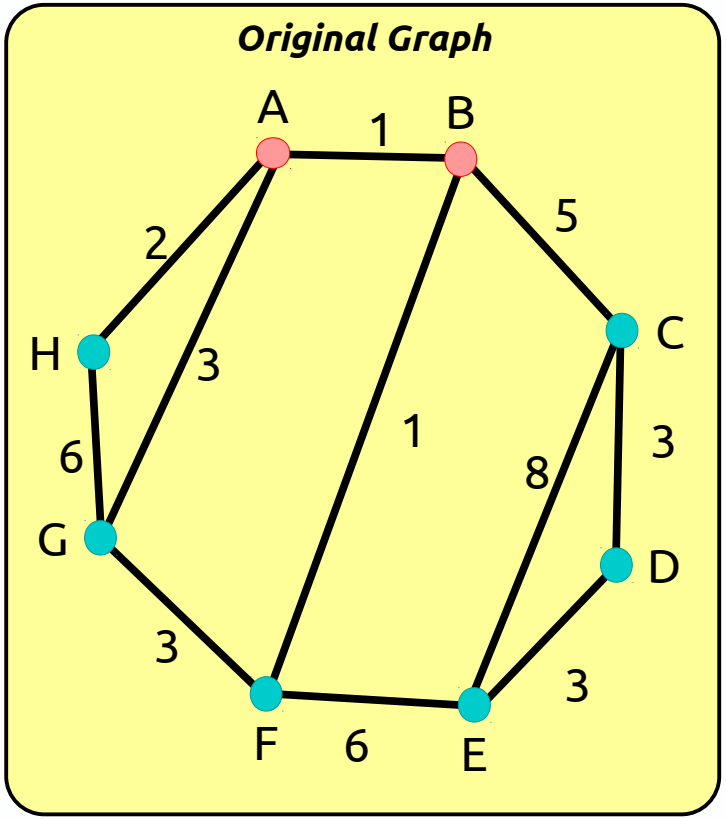

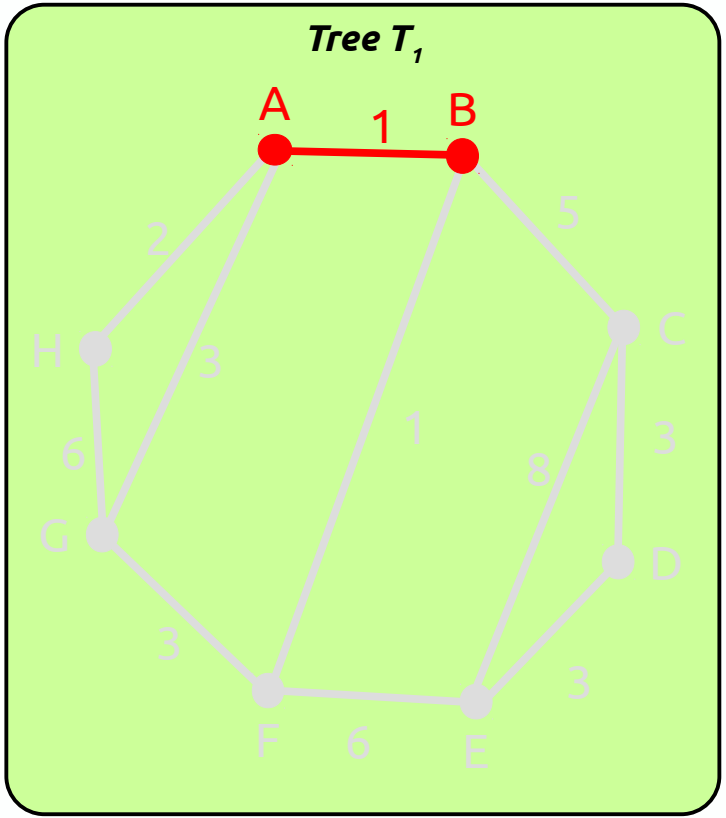

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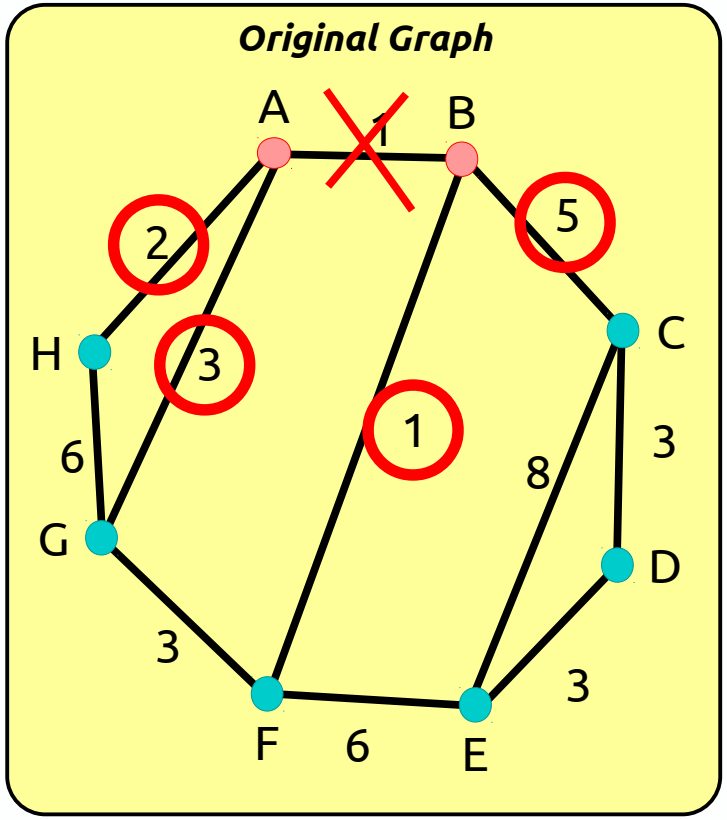
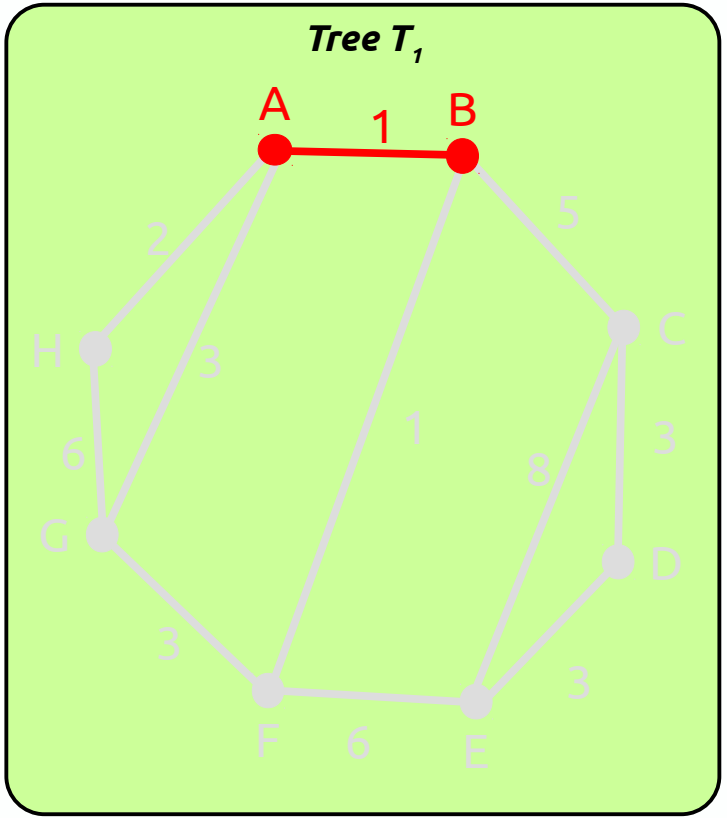
Output: The final result T_n is the tree returned by the algorithm.

3. Spanning Tree Algorithms

Next, we look at all edges connected to A and B in the original graph, and identify which has the smallest weight.





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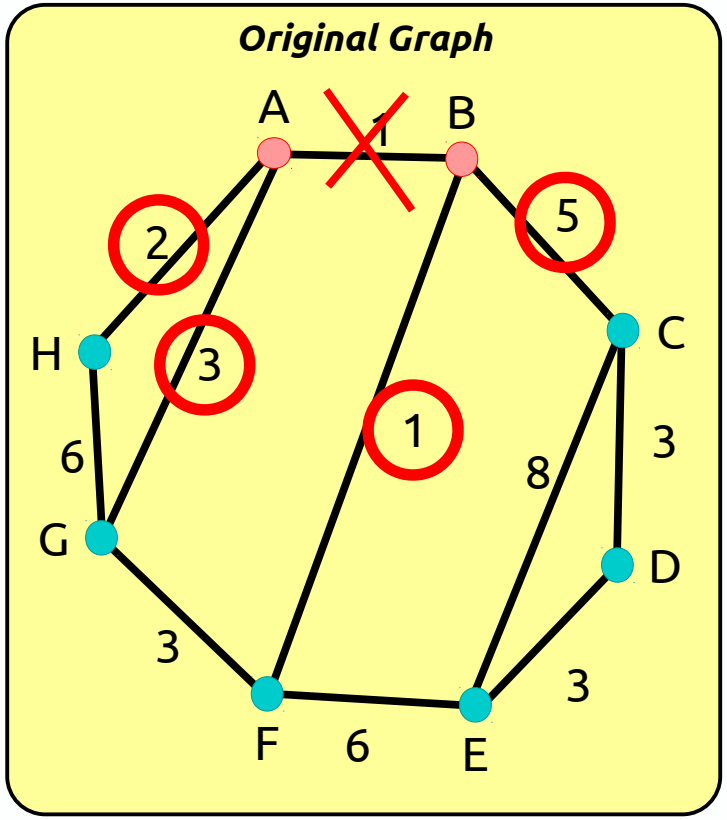
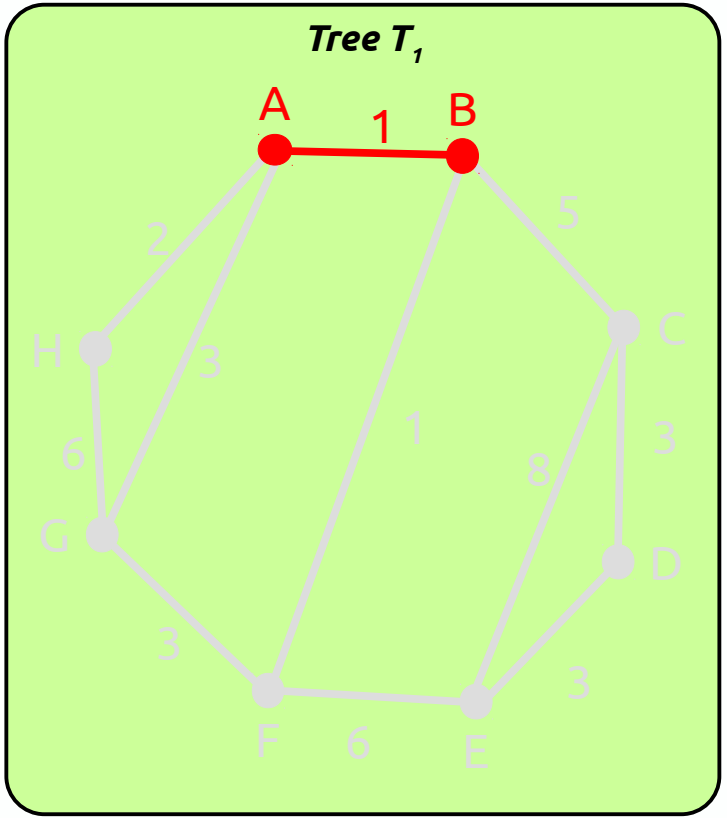
Output: The final result T_n is the tree returned by the algorithm.

3. Spanning Tree Algorithms

The next smallest edge is the connection from B to F, so we add that edge and F to the tree.





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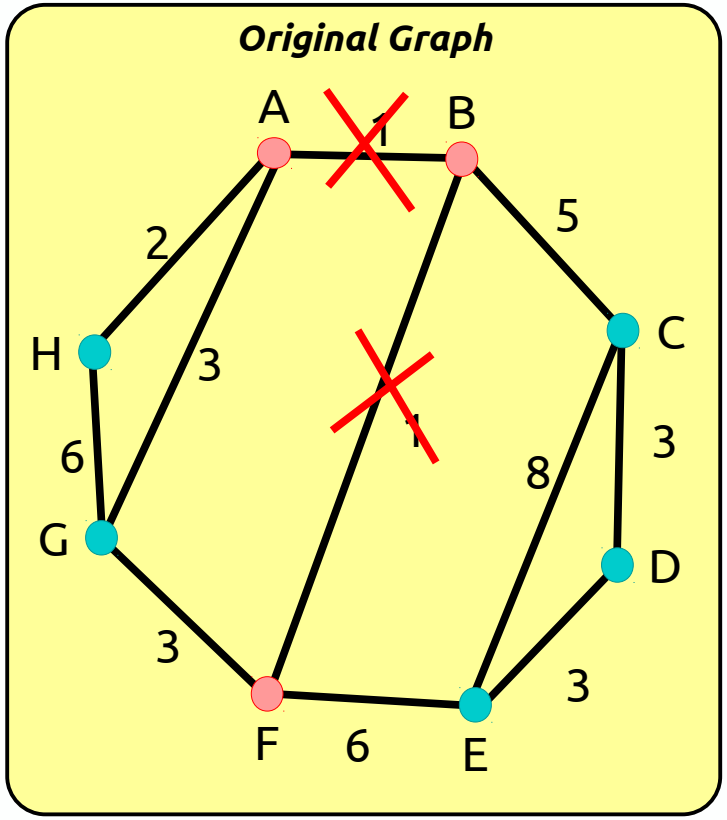

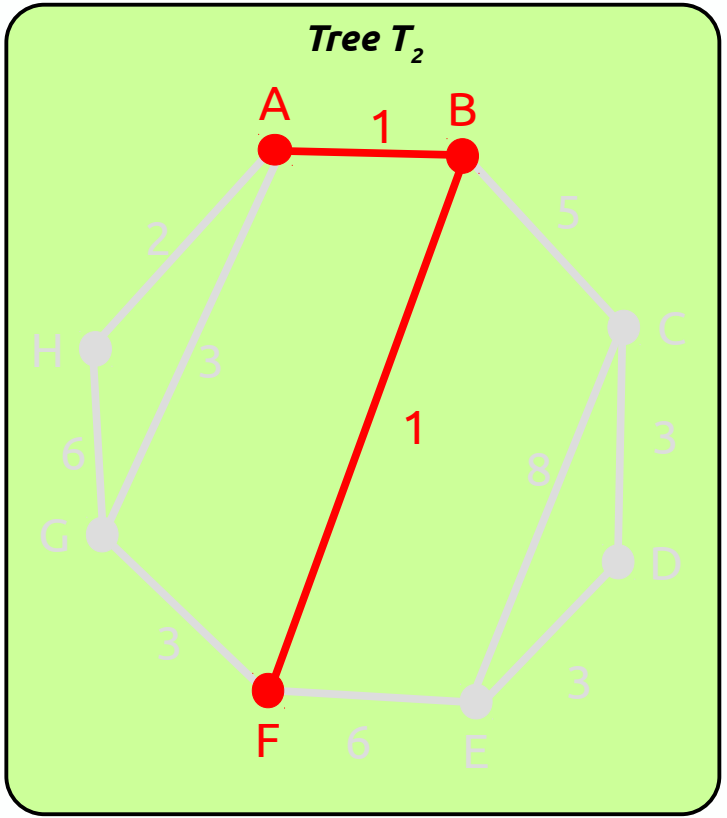

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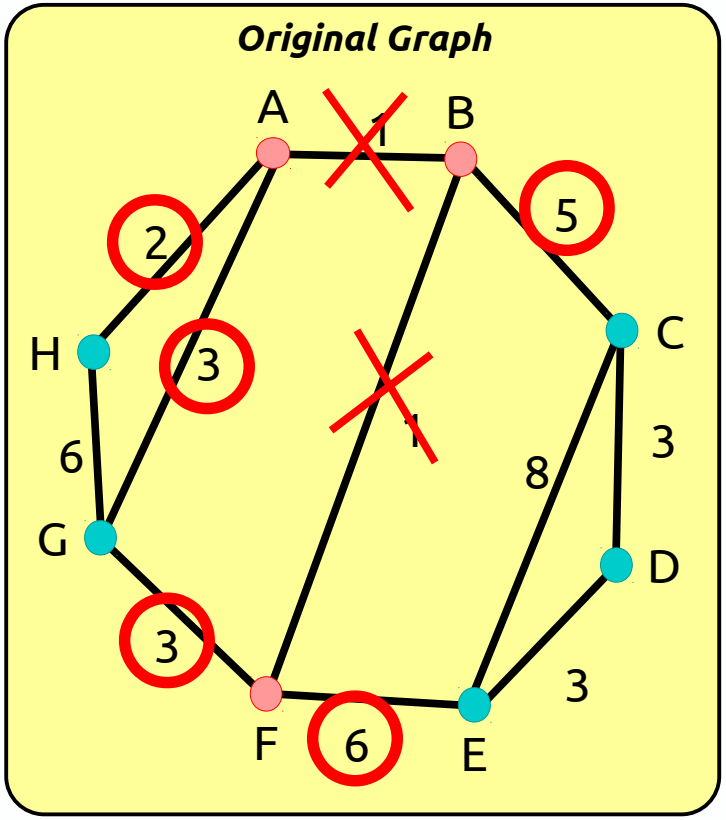
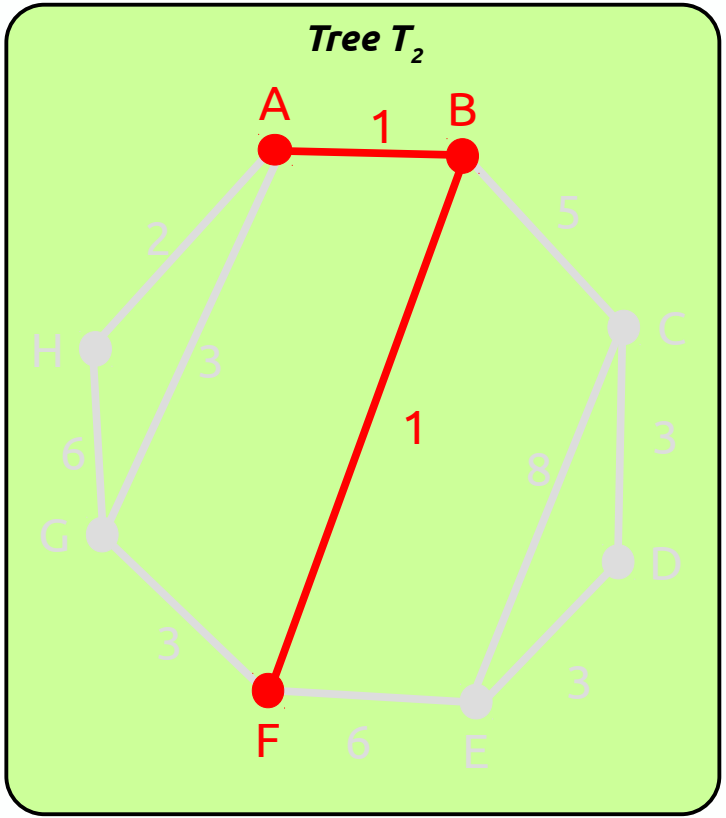
3. Spanning Tree Algorithms

And we continue…





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3. Spanning Tree Algorithms

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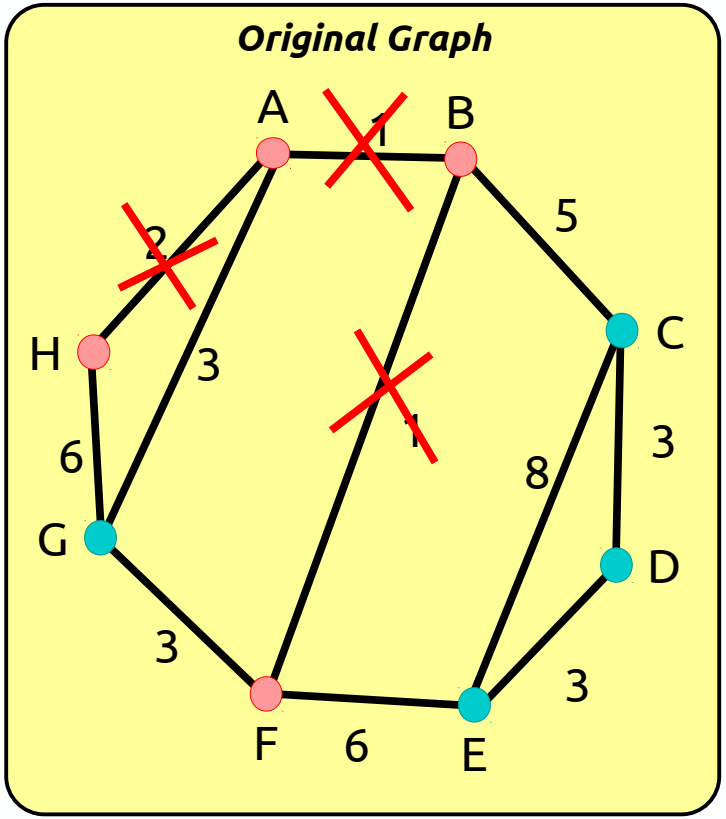

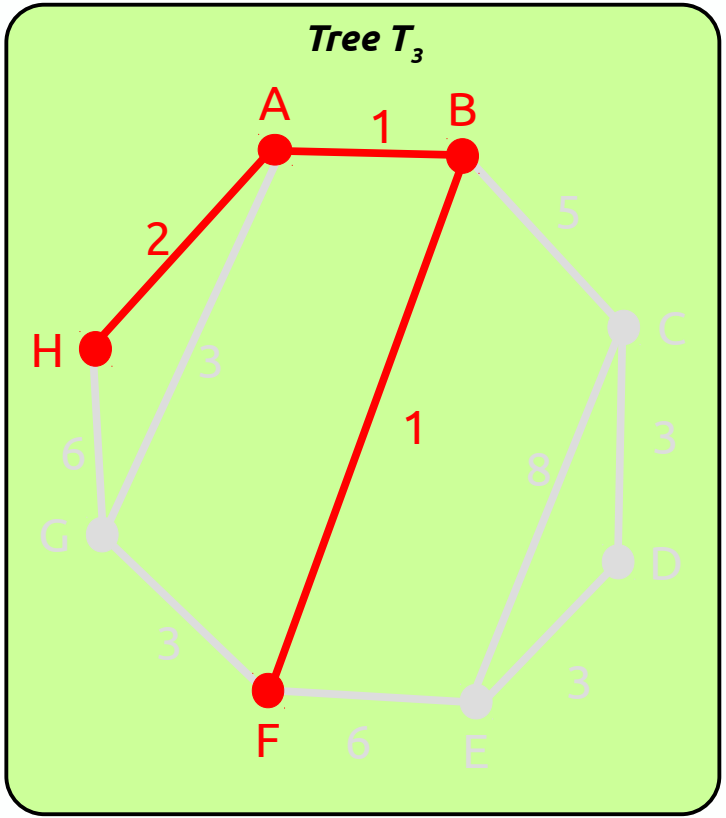

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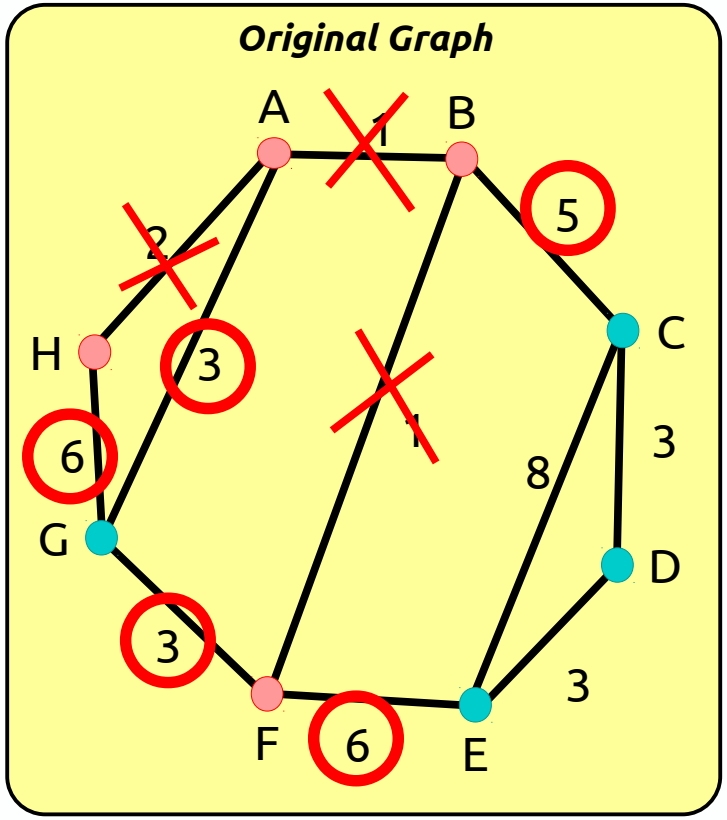
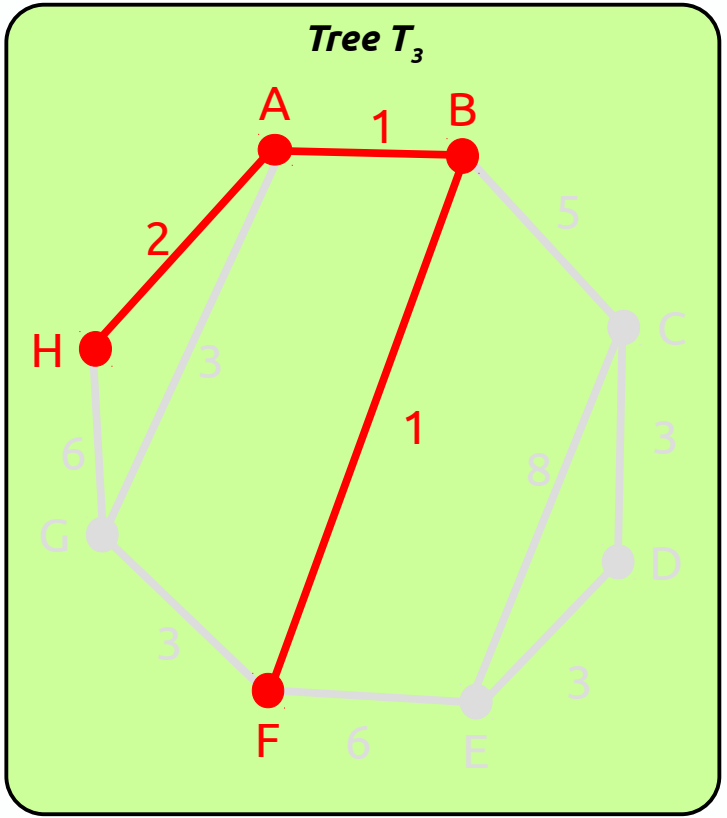
3. Spanning Tree Algorithms

We can choose either of the 3 edges, since they have the same weight.





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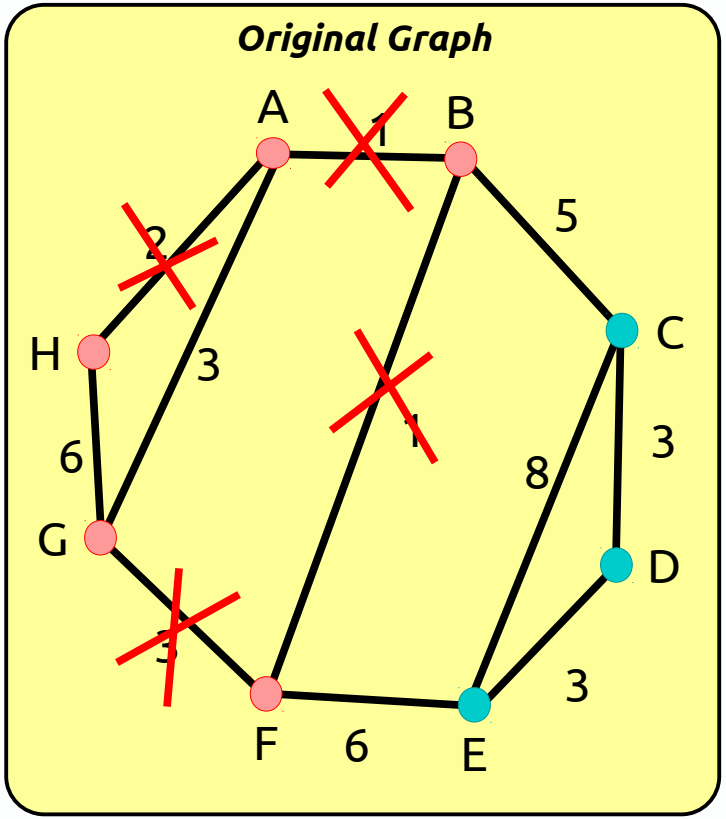
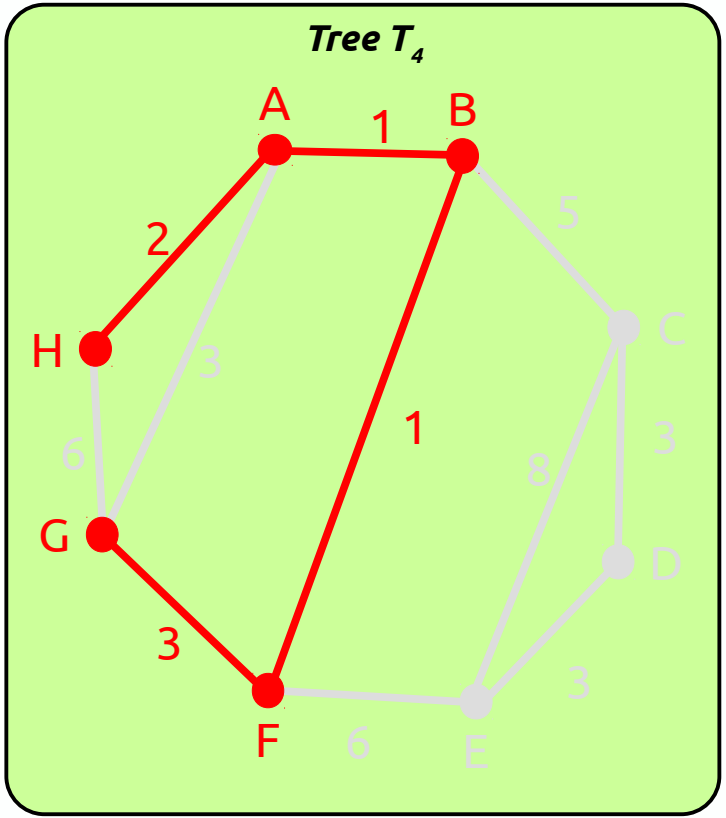
3. Spanning Tree Algorithms

And we continue…





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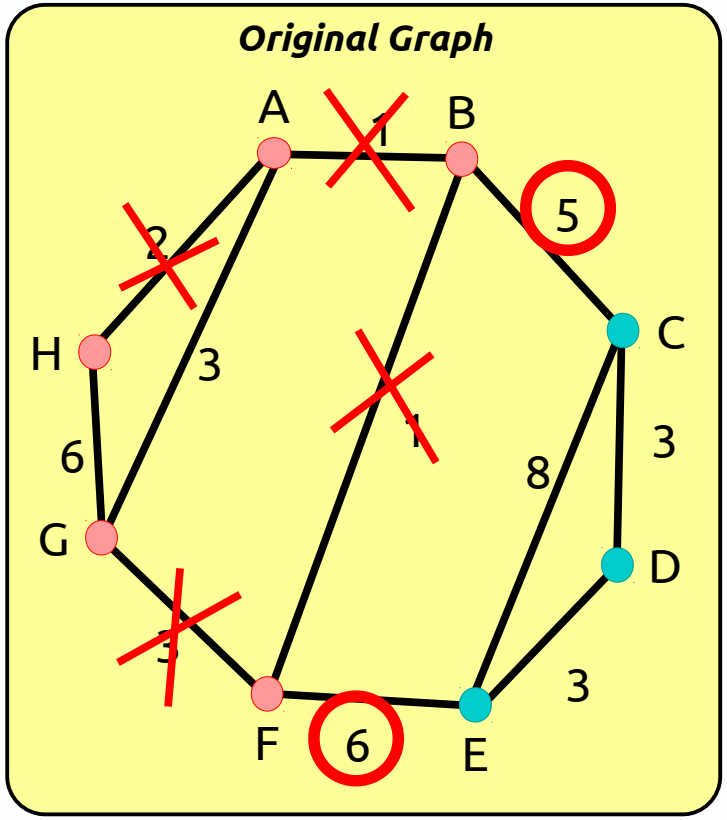
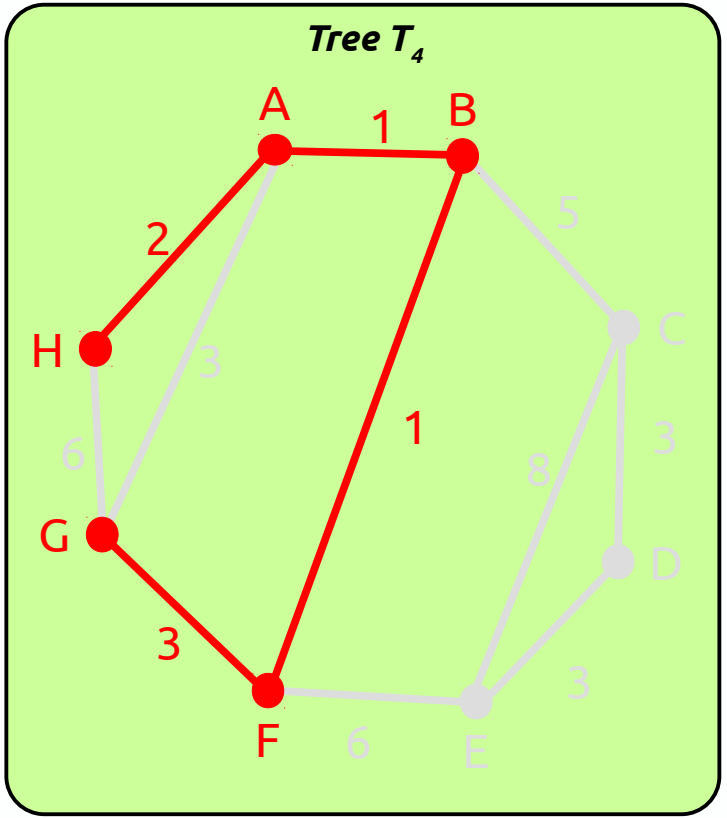
3. Spanning Tree Algorithms

We don't check any edges that are connecting nodes already in the tree.





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3. Spanning Tree Algorithms

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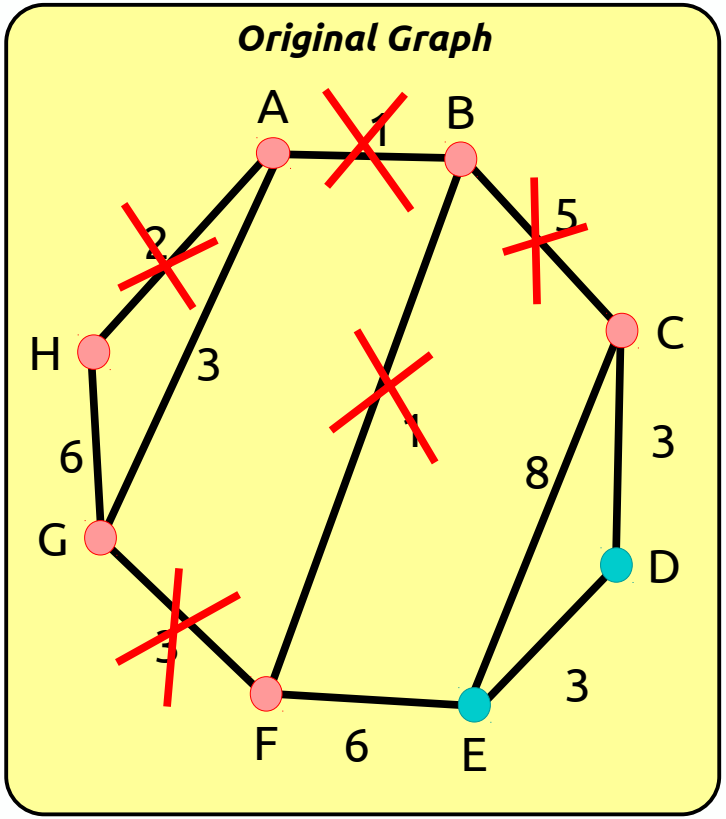

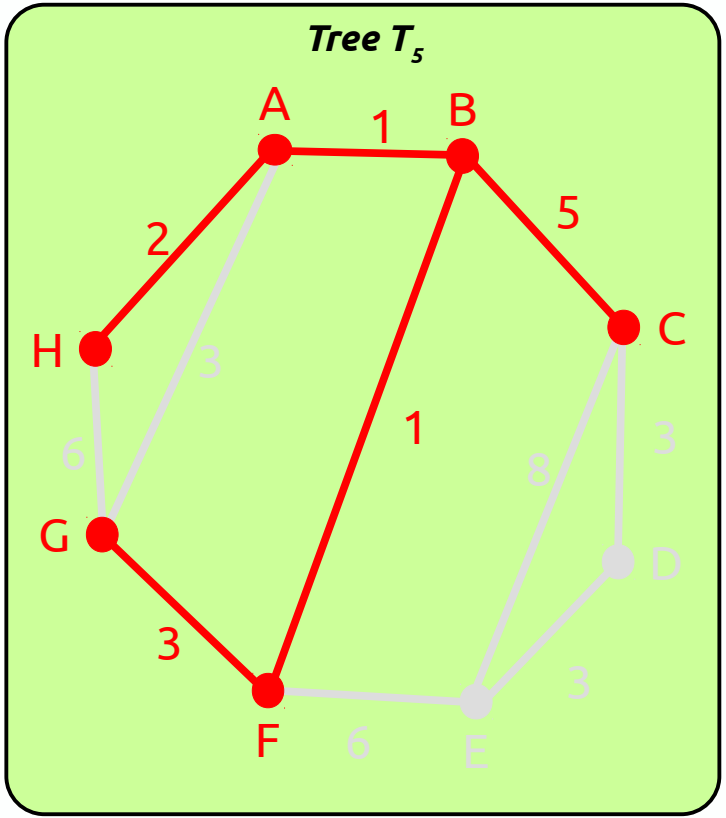

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3. Spanning Tree Algorithms

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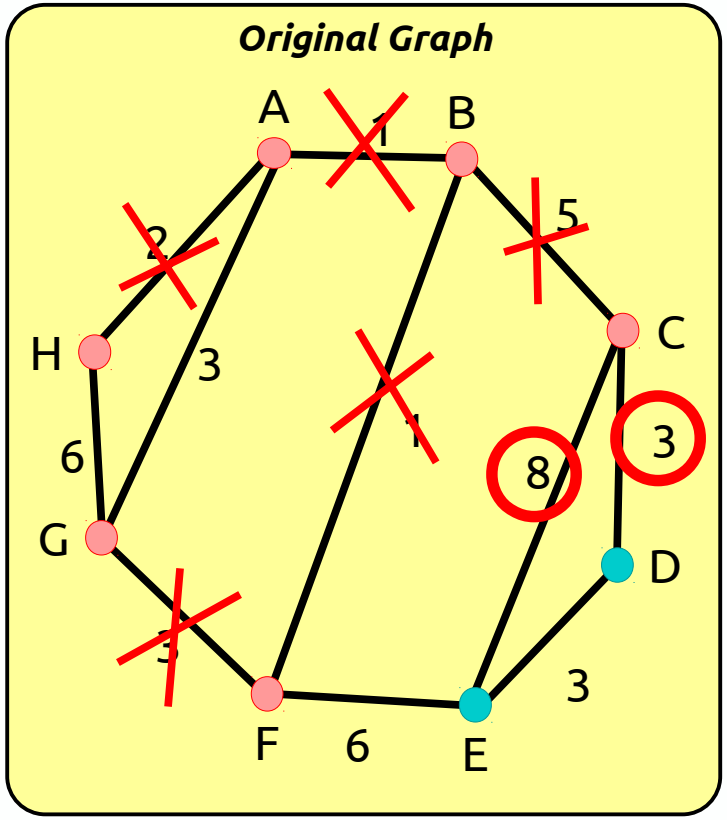

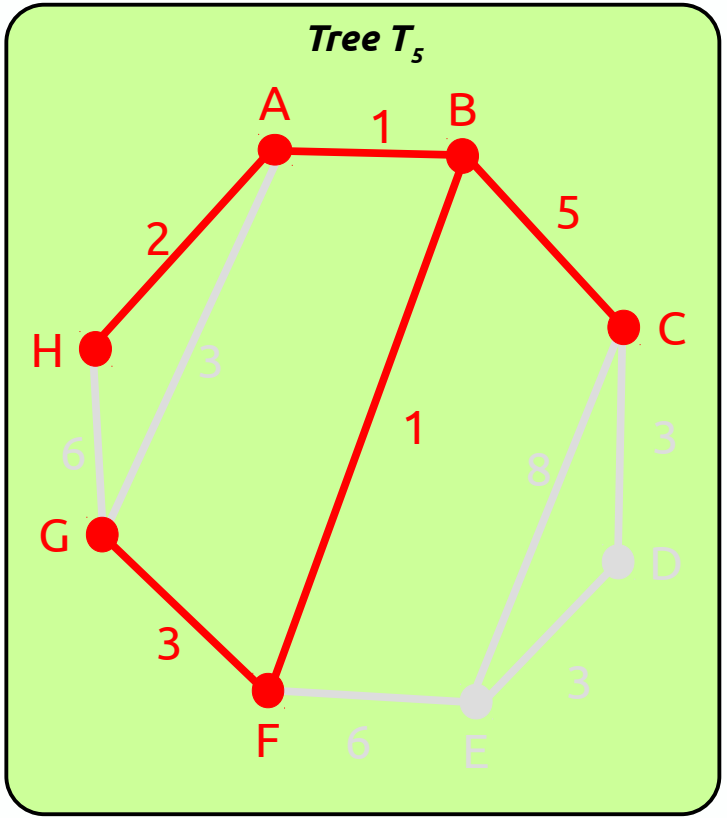

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3. Spanning Tree Algorithms

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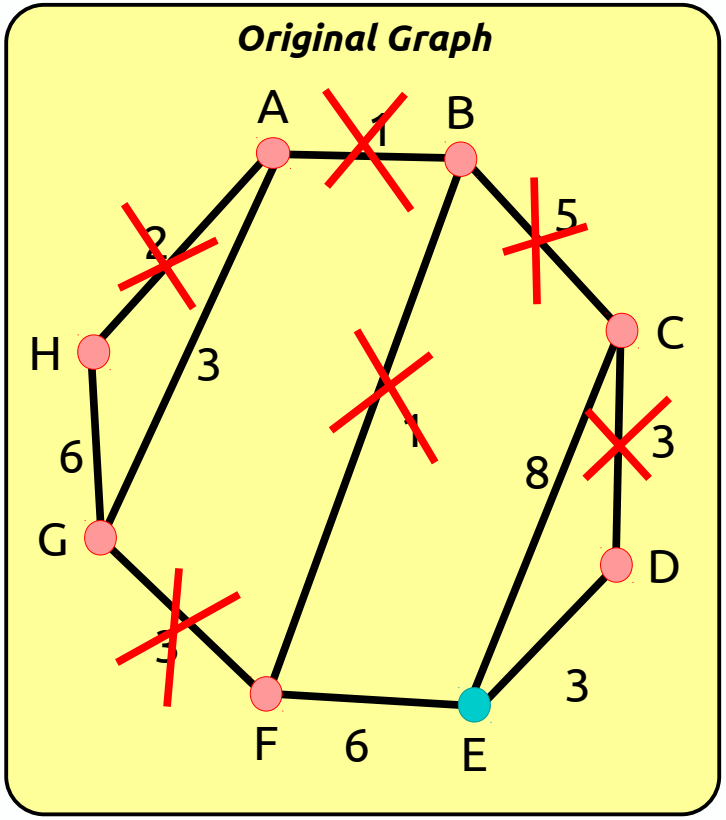

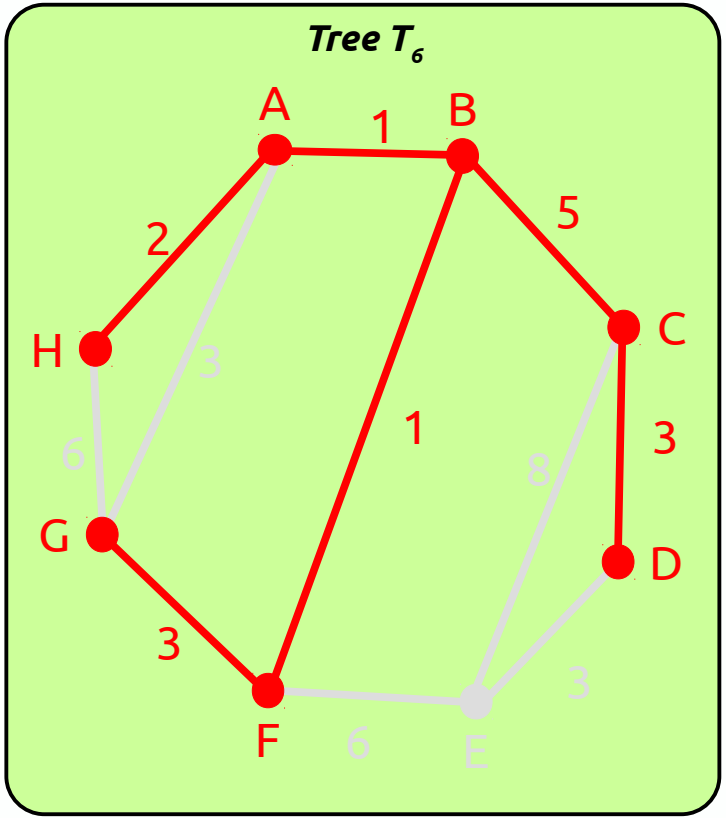

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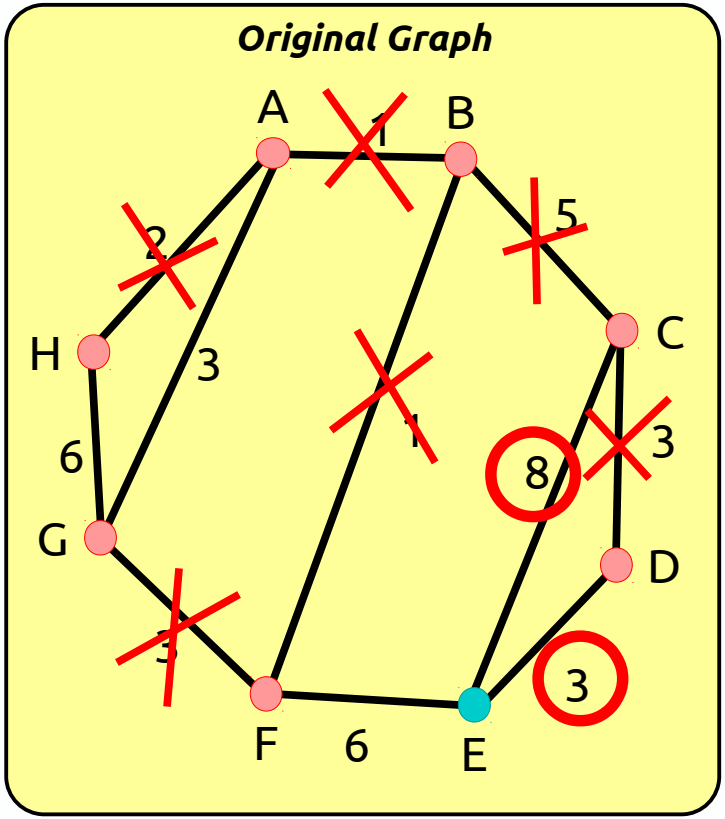

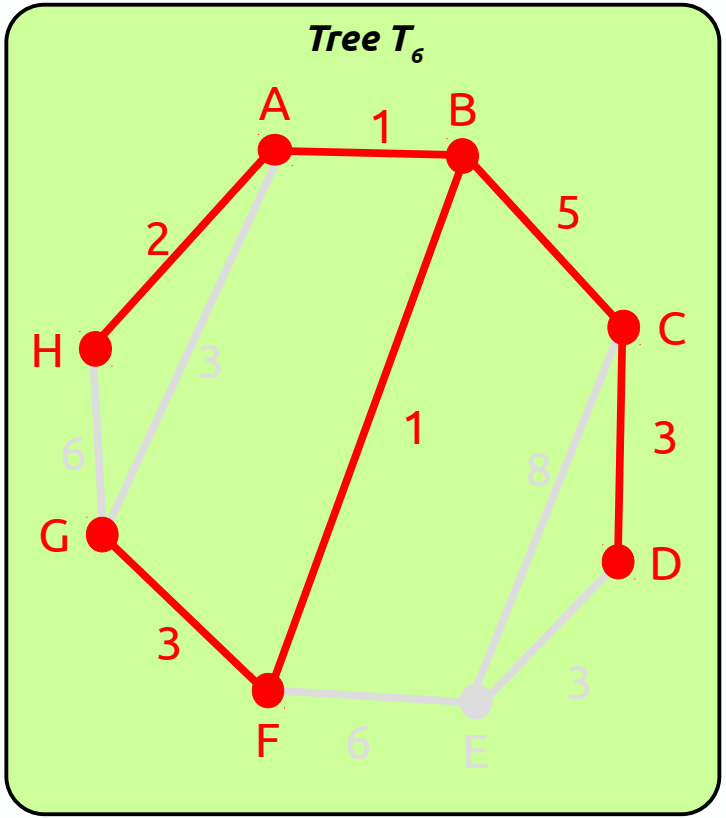

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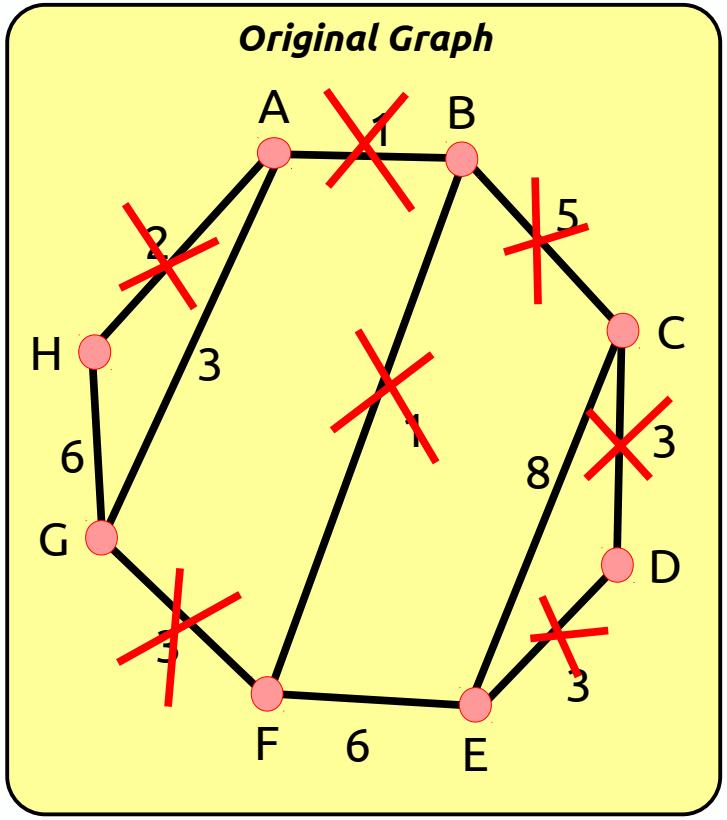
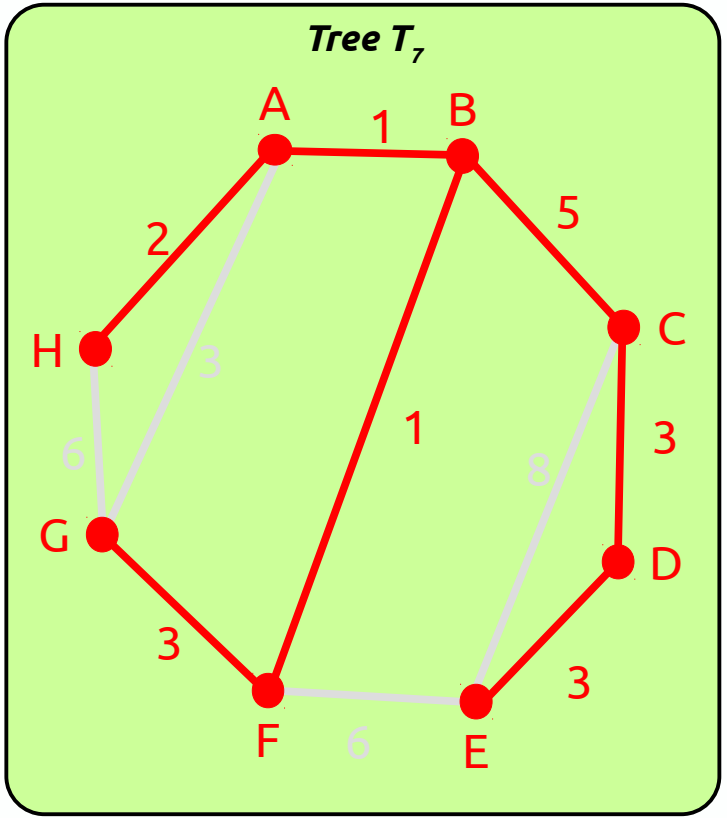
3. Spanning Tree Algorithms

And now we have covered all the nodes in the graph.





Notes

Prim's Spanning Tree Algorithm:

Input: A simple, connected graph G with $n + 1$ nodes.

Initialize: Let v_0 be any node in G, and let $T_0 = \{v_0\}$ be a tree with one node and no edges.

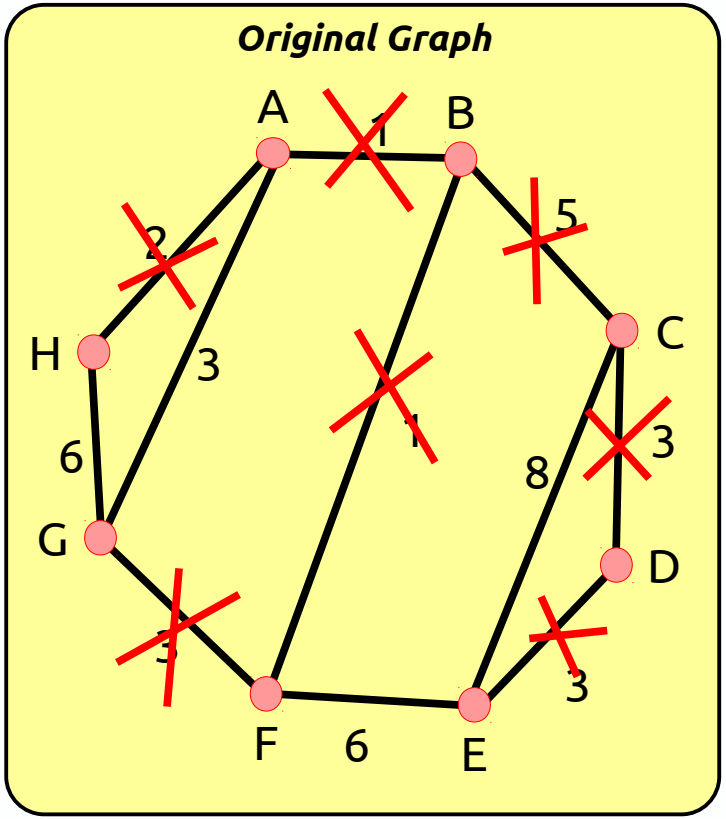
For each k from {1, 2, …, n}:

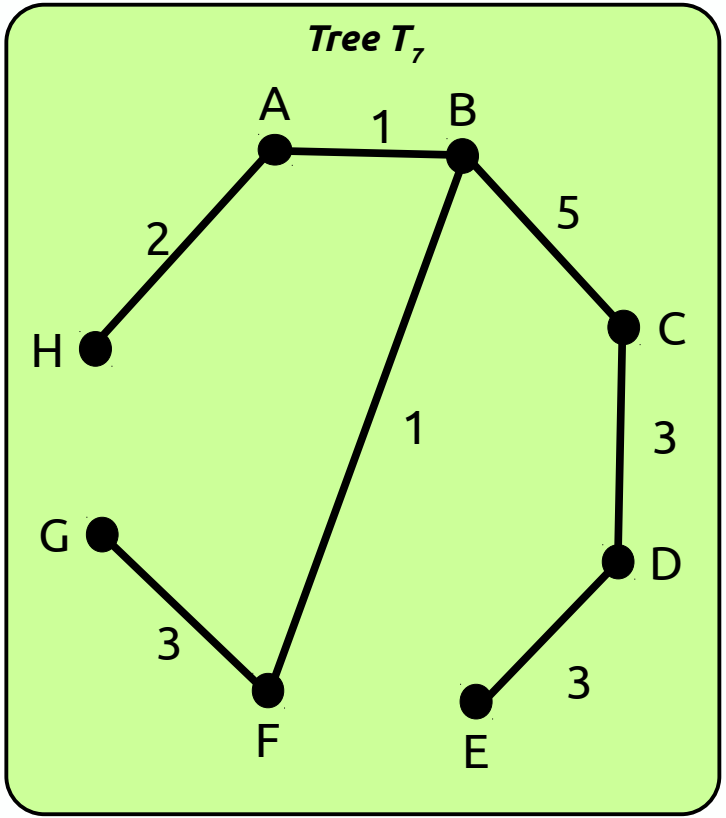
- Let $E_k = \{e$ an edge in G: e has one endpoint in T_{k-1} and the other endpoint not in $T_{k-1}\}$.
- Let e_k be the edge in E_k with the smallest weight. (In case of a tie, choose any edge of the smallest weight.)
- Let T_k be the tree obtained by adding edge e_k (along with its node not already in T_{k-1}) to T_{k-1}.

Output: The final result T_n is the tree returned by the algorithm.

3. Spanning Tree Algorithms

The result is a minimal spanning tree.





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Conclusion

Trees!