

# GRAPHS IN PUZZLES AND GAMES

# ABOUT

Here are some usages of graphs in riddle-like games to help solve the problem systematically.

# TOPICS

1. Wolves, Goats, & Cabbage
2. Water Jug Game

**WOLVES, GOATS,  
& CABBAGES**

# 1. WOLVES, GOATS, & CABBAGES



## Notes

### **Wolves, Goats, & Cabbages:**

- Can't leave wolf & goat together
- Can't leave goat & cabbage together

In the Wolves, Goats, and Cabbages game, you have to figure out how a traveller can get their Wolf, Goat, and Cabbage to the other side of a river in their boat.

The boat can only hold the traveller and one other thing.

The goat can't be left with the wolf; the wolf will eat it.

The cabbage can't be left with the goat; the goat will eat it.

# 1. WOLVES, GOATS, & CABBAGES

We can model this problem with a state diagram to help us solve it. Writing out the starting state, we have:

## State 1

Island 1: Traveller, Wolf, Goat, Cabbage  
Island 2: Nothing

Or to save space, we can shorten it:

## State 1

(T,W,G,C,  $\emptyset$ )



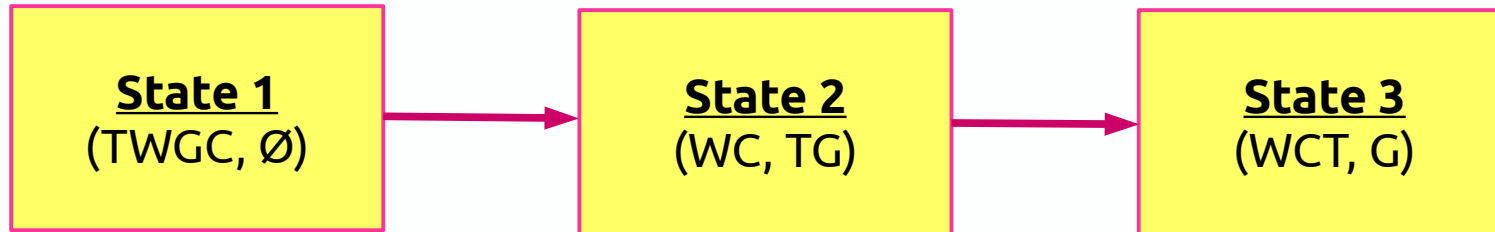
## Notes

### **Wolves, Goats, & Cabbages:**

- Can't leave wolf & goat together
- Can't leave goat & cabbage together

# 1. WOLVES, GOATS, & CABBAGES

Where can we go from this state? Well, we can't leave the wolf+goat alone, or the goat+cabbage alone, so we have to take the goat.



From there, we can either go *back* to the original state, or leave the cabbage there and go back to the first island.

## Notes

### **Wolves, Goats, & Cabbages:**

- Can't leave wolf & goat together
- Can't leave goat & cabbage together

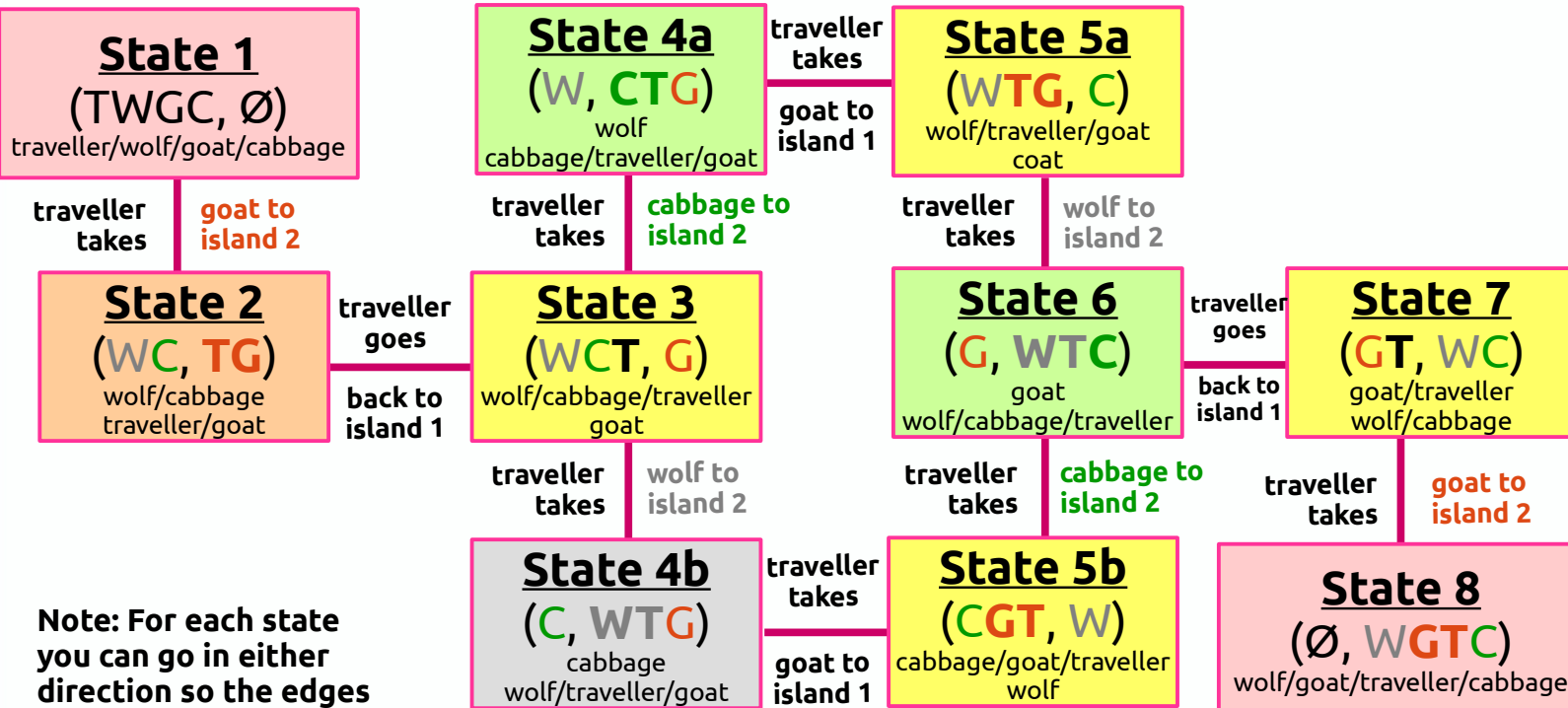
# 1. WOLVES, GOATS, & CABBAGES

And we can keep drawing out each state, one by one.

## Notes

### Wolves, Goats, & Cabbages:

- Can't leave wolf & goat together
- Can't leave goat & cabbage together



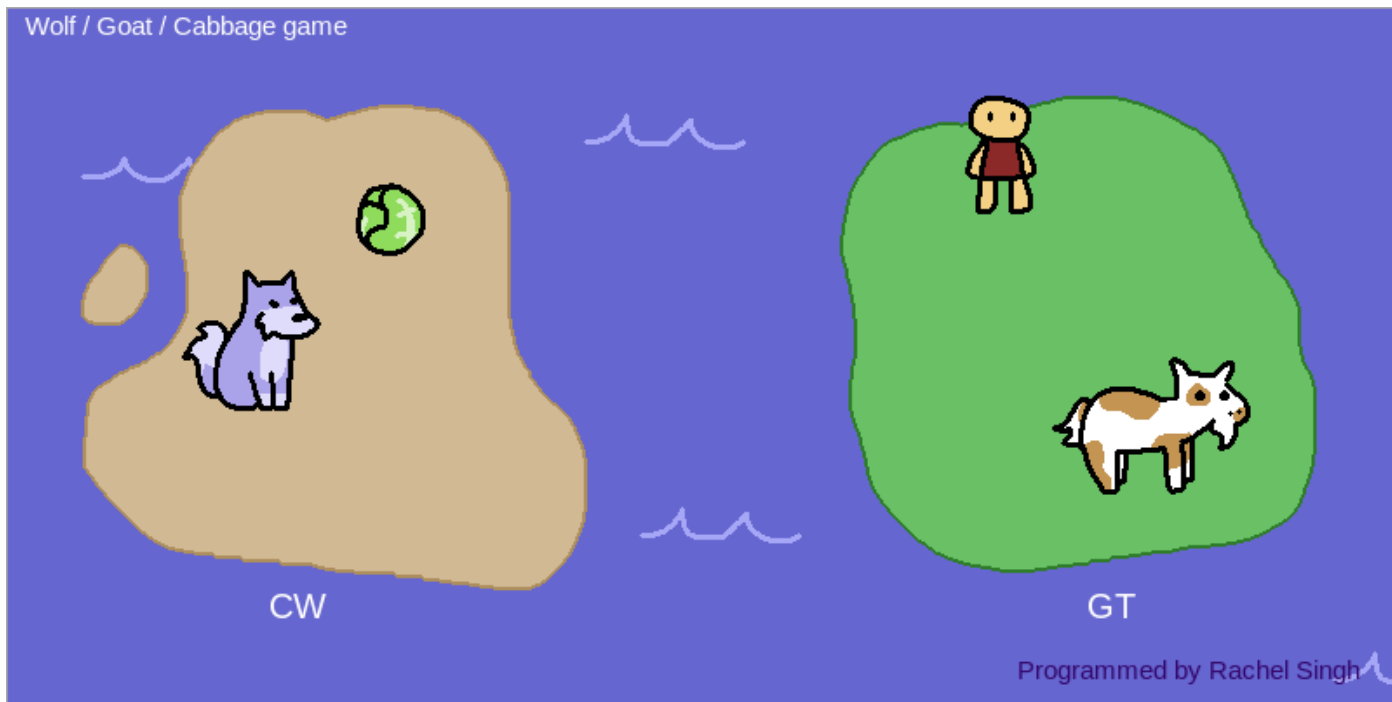
Note: For each state you can go in either direction so the edges don't have arrows.



# 1. WOLVES, GOATS, & CABBAGES

Can work through these states graphically here:

<https://rachels-courses.github.io/Visualizations/Discrete-Math/Cabbages/cabbages.html>



## Notes

### Wolves, Goats, & Cabbages:

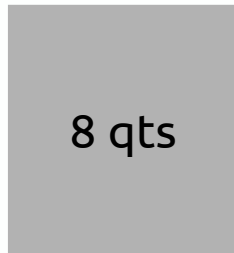
- Can't leave wolf & goat together
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# WATER JUG GAME

## 2. WATER JUG GAME

In the Water Jug Game, there is some amount of water that starts out in a pail. Using only two jugs of certain sizes, you have to measure some amount of water.

In the book, the pail holds 8 qts, and the two jars hold 5 and 3 qts. The challenge is to measure so that 4 qts are in the large jug, and 4 qts are in the pail.



Notes

## 2. WATER JUG GAME

The first state is for all the water to be in the pail.

### State 1

Pail: 8 qts  
5q Jug: 0 qts  
3q Jug: 0 qts

Or to save space, we can shorten it:

### State 1

(8,0,0)

Notes

# 2. WATER JUG GAME

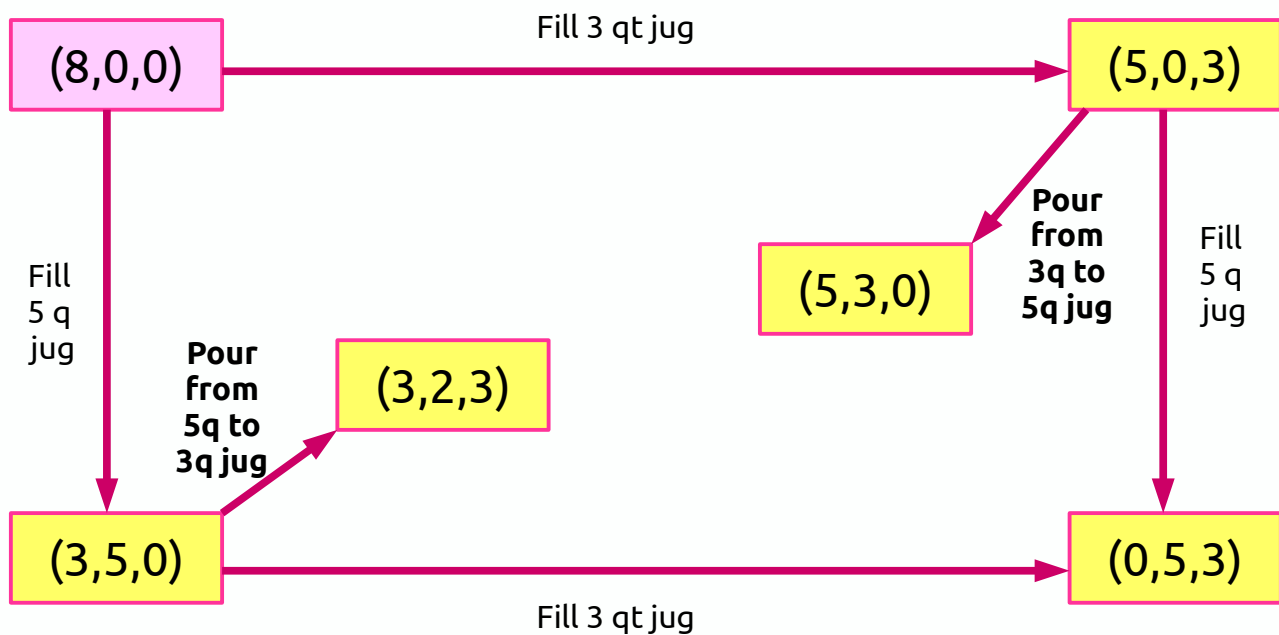
For the first move, we can take out 5 qts, or take out 3 qts.



Notes

# 2. WATER JUG GAME

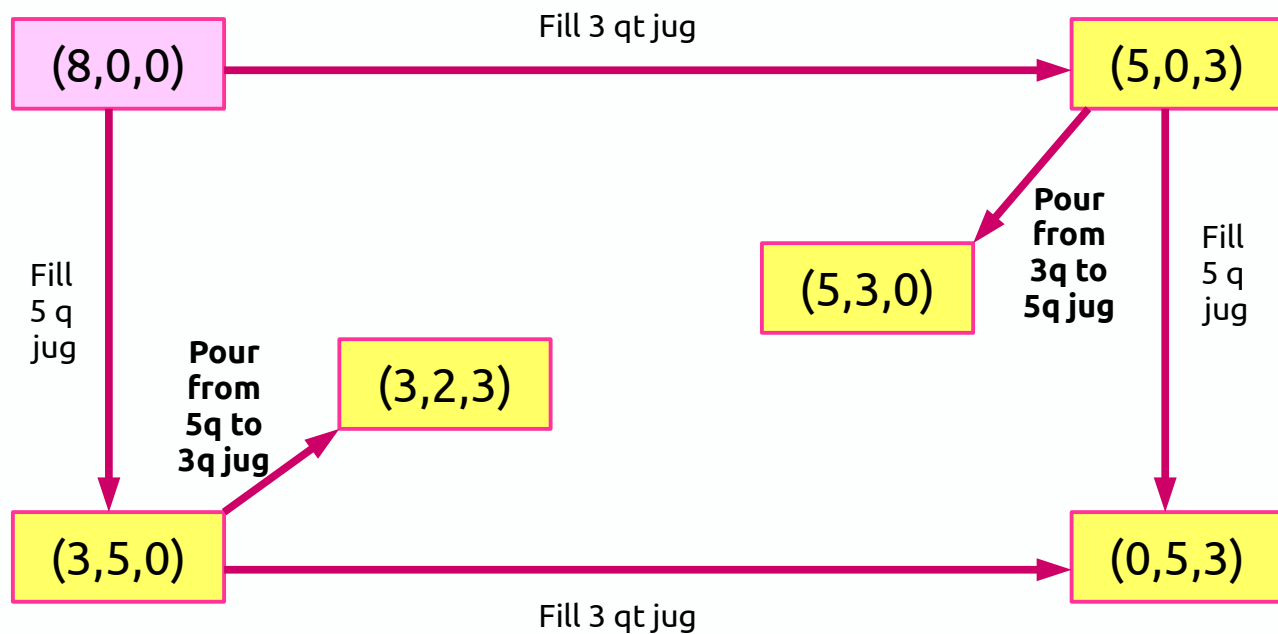
These two states also have 2 choices of moves each.



Notes

# 2. WATER JUG GAME

As we keep going, the challenge is to get to the state where we have (4,4,0) – 4 quarts in the pail and 4 quarts in the big jug. Can you figure out the diagram?



Notes

# CONCLUSION

**Trees!**