Hamiltonian Cycles and The Travelling Salesperson Problem

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About

Trees are a handy structure in Data Structures, and are also a part of Graph Theory.

Topics

1. Seven Bridges of Königsberg

2. Hamiltonian Cycles

3. Travelling Salesperson

Seven Bridges of Königsberg

1. Seven Bridges

The Seven Bridges of Königsberg is a historically notable problem in mathematics.

The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other, or to the two mainland portions of the city, by seven bridges. The problem was to devise a walk through the city that would cross each of those bridges once and only once.

From https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

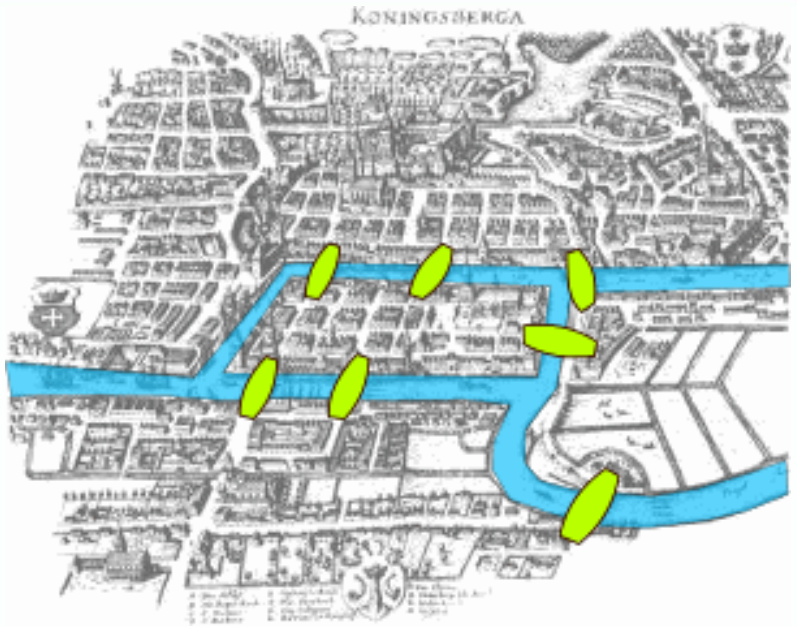
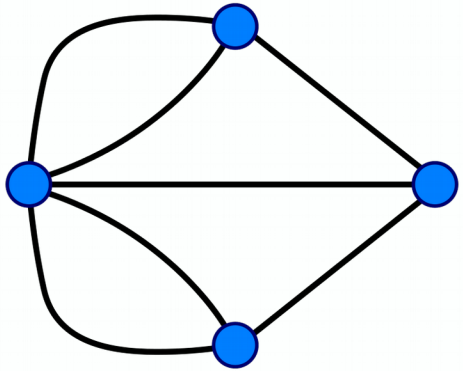


Image: The problem of the Seven Bridges of Königsberg, public domain image from https://en.wikipedia.org/wiki/File:Konigsberg_bridges.png

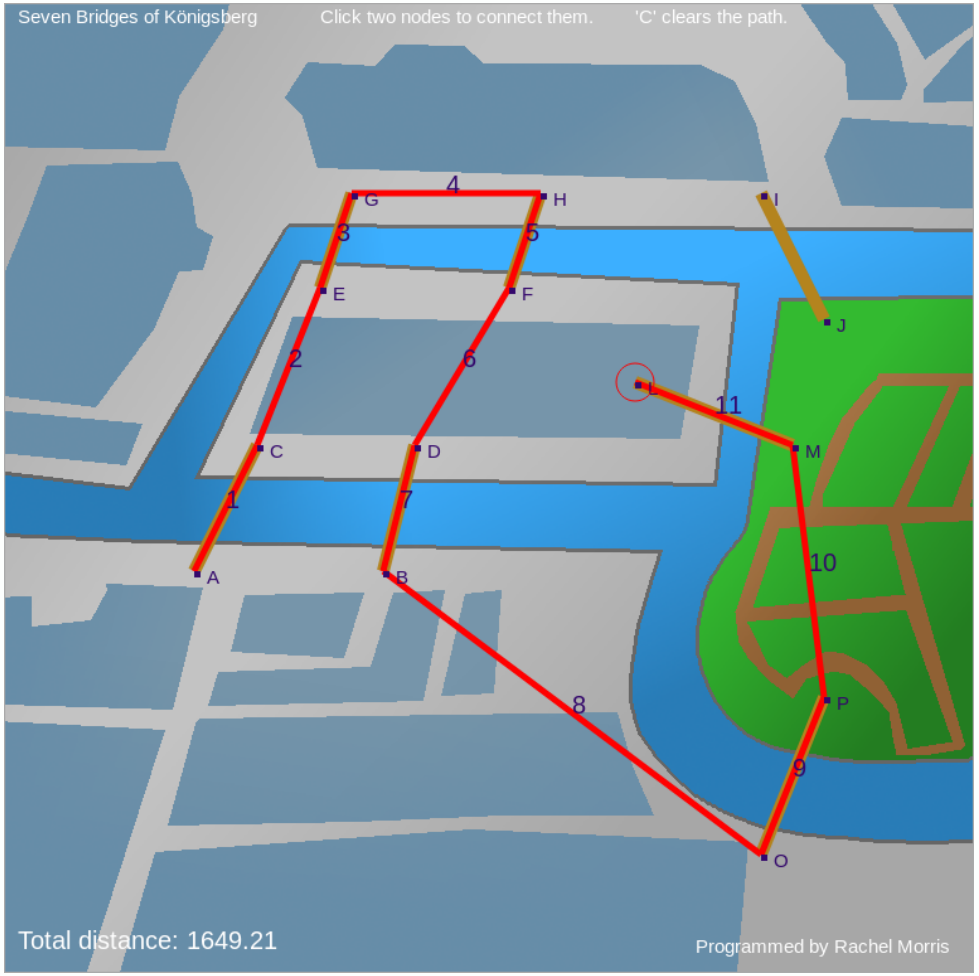
1. Seven Bridges

Bridges visualization:
https://rachels-courses.github.io/Visualizations/Discrete-Math/Seven-Bridges/bridges.html

If we generalize this to think of each of the landmasses as a “node”, and the bridges as “edges”, we can draw the graph to represent this problem:

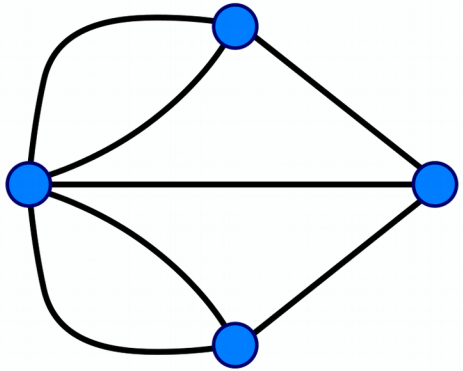


From https://en.wikipedia.org/wiki/File:K%C3%B6nigsberg_graph.svg , under a CC Attribution ShareAlike license, image by Booyabazooka



1. Seven Bridges

To be able to traverse every bridge exactly once, there needs to be an even degree for each of the nodes, but that is not the case here, so it is impossible to do so.



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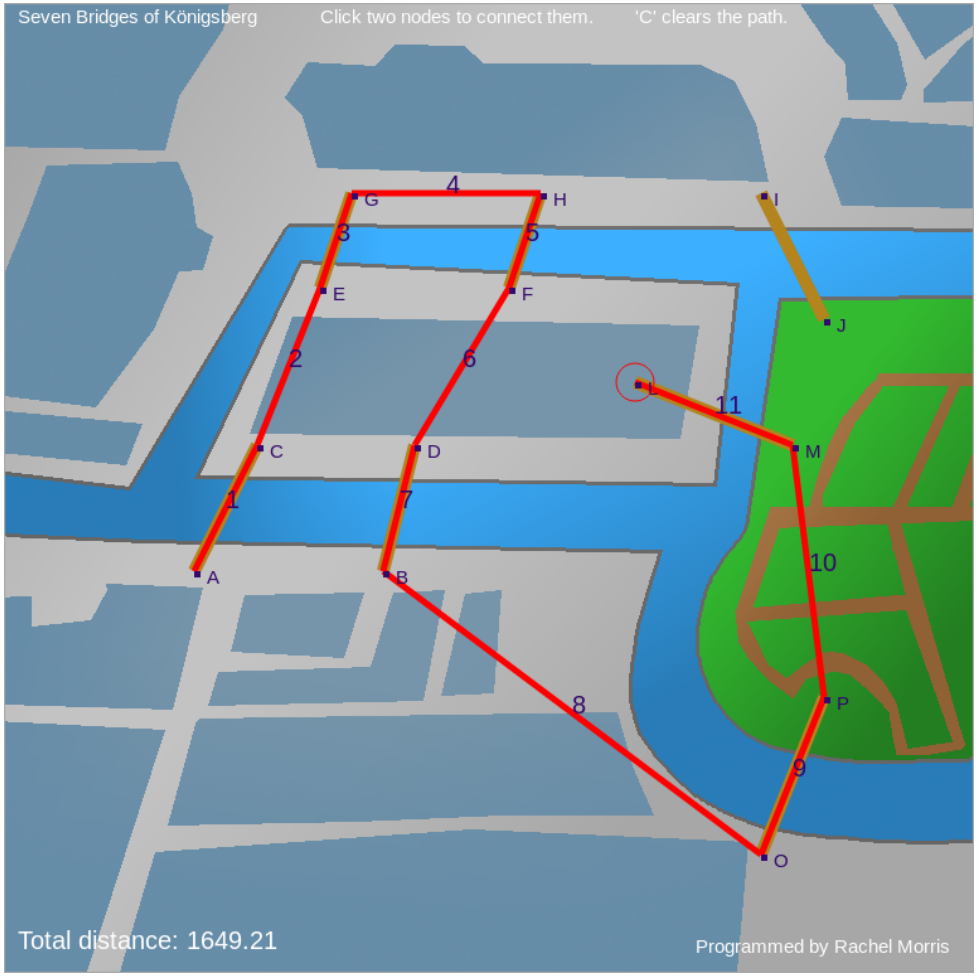
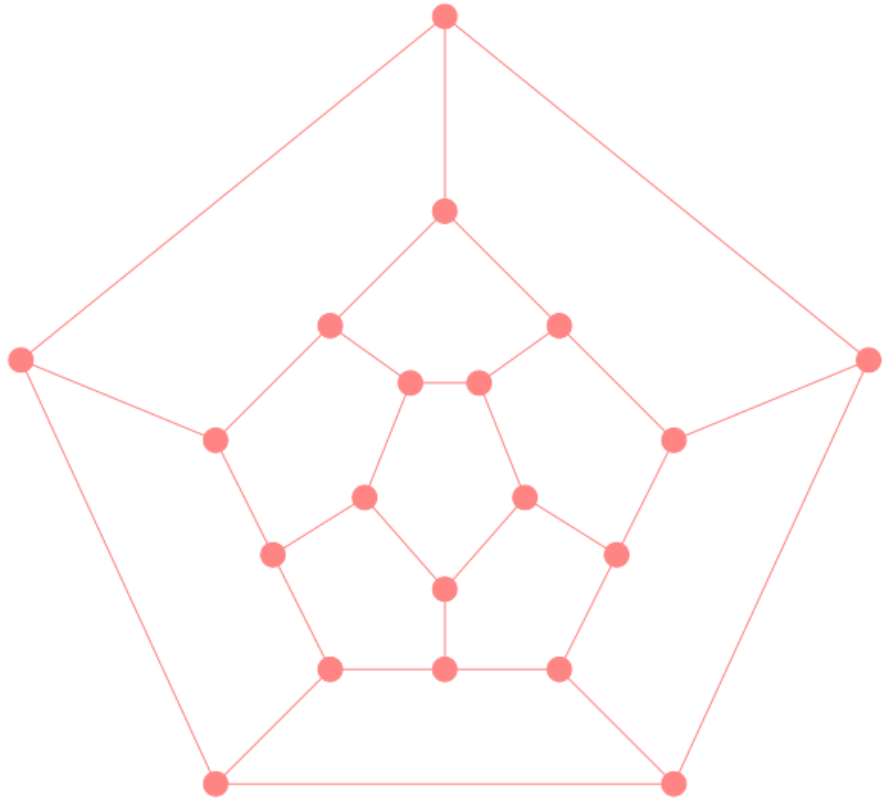


Hamiltonian Cycles

2. Hamiltonian Cycles

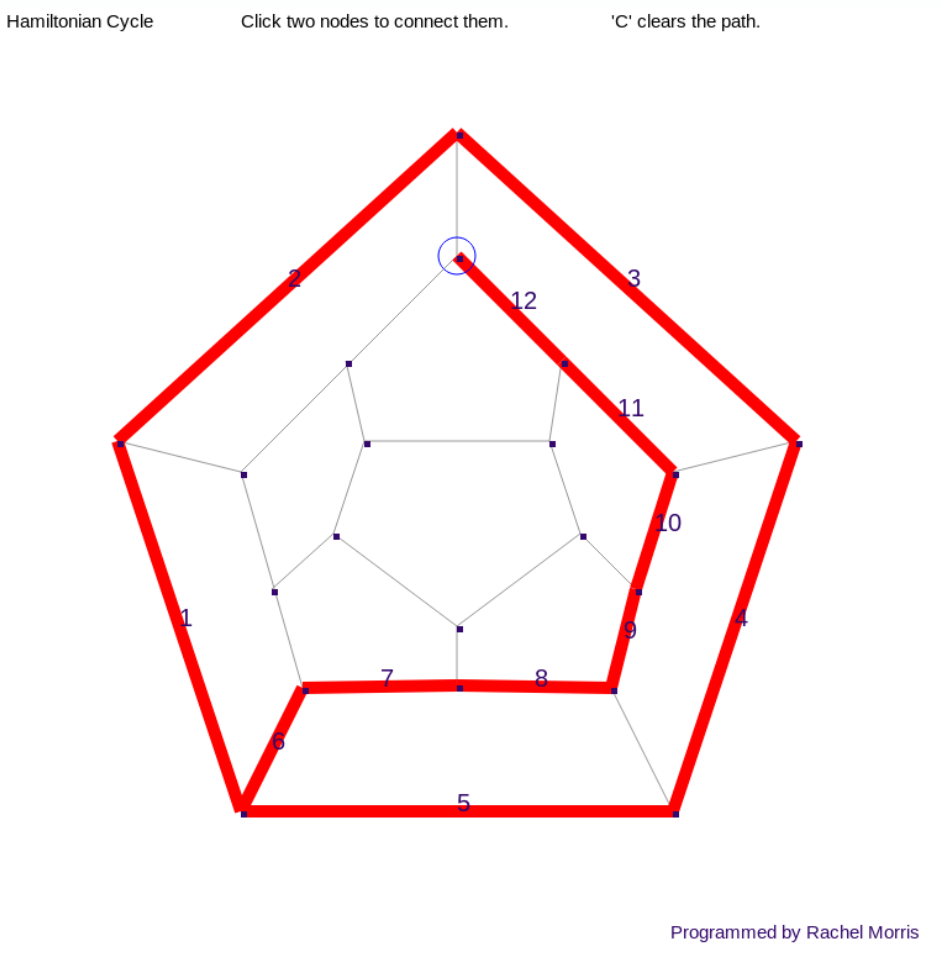
In the mathematical field of graph theory the Hamiltonian path problem and the Hamiltonian cycle problem are problems of determining whether a Hamiltonian path (a path in an undirected or directed graph that visits each vertex exactly once) or a Hamiltonian cycle exists in a given graph (whether directed or undirected).

https://en.wikipedia.org/wiki/Hamiltonian_path_problem



2. Hamiltonian Cycles

Visualization:
https://rachels-courses.github.io/Visualizations/Discrete-Math/Hamiltonian-Cycle/hamiltonian.html



The Travelling Salesperson Problem

3. The Travelling Salesperson Problem

"Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?"

https://en.wikipedia.org/wiki/Travelling_salesperson_problem

3. The Travelling Salesperson Problem

Visualization:
https://rachels-courses.github.io/Visualizations/Discrete-Math/Travelling-Salesperson/salesperson.html

Conclusion

This is just a small extra section in our discrete math textbook. You will probably hear of these problems later on in CS work.