Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own "dropbox" on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

1.2: Propositions

1. Determine which of the following statements are propositions:
    a. I am a student.
    b. How many students are in the class?
    c. Run!
2. Write the negation of each of the following:
    a. $x > 10$
    b. $y = 10$
    c. $x \in S$
    d. $p \wedge q$
    e. $r \vee s$
3. What is the result of each of the following statements?
    a. $p \wedge \neg q$ when p is true and q is true.
    b. $p \vee \neg q$ when p is false and q is false.
    c. $\neg(p \vee q)$ when p is false and q is false.

4. Translate the following statements into propositional logic using the given variables:

   b: The sandwich has bacon l: The sandwich has lettuce
   t: The sandwich has tomato c: The sandwich has cheese

   a. The sandwich has bacon, lettuce, and tomato.
   b. The sandwich has tomato and it doesn't have cheese.
   c. The sandwich has tomato, but it doesn't have neither bacon nor cheese.
   d. Either the sandwich has (only) lettuce and tomato, or it has (only) bacon and cheese.

5. Using the propositional variables, p, q, and r, write compound statements that meets each of the given criteria.

   a. p and q are true, but not r.
   b. p and one other statement are true, but not all three.
   c. Exactly two statements are true, but not only one and not all three.

6. Complete the truth tables for the following statements:

   a. $p \wedge \neg q$
   b. $\neg p \vee q$
   c. $(p \wedge q) \vee (p \wedge r)$
   d. $p \wedge \neg(q \vee r)$
   e. $(p \vee q) \vee (\neg p \vee \neg q)$
   f. $p \wedge \neg p$

Propositions - Answer key

1. a. Proposition
   b. Not a proposition
   c. Not a proposition

2. a. $\neg(x > 10) \equiv x \leq 10$
   b. $\neg(y = 10) \equiv y \neq 10$
   c. $\neg(x \in S) \equiv x \notin S$
   d. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
   e. $\neg(r \vee s) \equiv \neg r \wedge \neg s$

3. a. $p \wedge \neg q$ when p is true and q is true.
   true and NOT(true) $\equiv$ true and false $\equiv$ false.
   b. $p \vee \neg q$ when p is false and q is false.
   false or NOT(false) $\equiv$ false or true $\equiv$ true.
   c. $\neg(p \vee q)$ when p is false and q is false.
   NOT(false or false) $\equiv$ NOT (false) $\equiv$ true.

4. Statements can be written in various ways. Check your correctness by validating with a truth table.
   a. The sandwich has bacon, lettuce, and tomato.
   $b \wedge l \wedge t$
   b. The sandwich has tomato and it doesn't have cheese.
   $t \wedge \neg c$
   c. The sandwich has tomato, but it doesn't have neither bacon nor cheese.

   $t \wedge \neg b \wedge \neg c$ or $t \wedge \neg(b \vee c)$

   d. Either the sandwich has (only) lettuce and tomato, or it has (only) bacon and cheese.
   $(l \wedge t \wedge \neg b \wedge \neg c) \vee (b \wedge c \wedge \neg l \wedge \neg t)$

5. These can be tricky, so it can be useful to check with a truth table. Statements can be written in other forms as well.

   a. p and q are true, but not r.
   $p \wedge q \wedge \neg r$

   b. p and one other statement are true, but not all three.
   $p \wedge (q \vee r) \wedge \neg(p \wedge q \wedge r)$ or
   $(p \wedge q \wedge \neg r) \vee (p \wedge r \wedge \neg q)$

   c. Exactly two statements are true, but not only one and not all three.
   $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

6. a. $p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

   b. $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

   c. $(p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

   Notice that you can check your answer on the left; p needs to be true, and either q or r need to also be true. The only states that result in true are the ones with TTT, TTF, and TFT.

d. $p \wedge \neg(q \vee r)$

p	q	r	$q \vee r$	$\neg(q \vee r)$	$p \wedge \neg(q \vee r)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

e. $(p \vee q) \vee (\neg p \vee \neg q)$

p	q	$p \vee q$	$\neg p \vee \neg q$	$(p \vee q) \vee (\neg p \vee \neg q)$
T	T	T	F	T
T	F	T	T	T
F	T	T	T	T
F	F	F	T	T

Here we have a tautology - all states lead to a True result.

f. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

This is a contradiction - we cannot have p be both true and false at the same time, so the result of $p \wedge \neg p$ is false for all states.