

Discrete Structures I: Logic: Implications

Textbooks: Ensley & Crawley: Chapter 1.5

Johnsonbaugh: Chapter 1.3

Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own “dropbox” on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

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Implications (Conditional Propositions)

1. For each of these implications, define variables for each proposition and rewrite the statement symbolically.
 - a. “If it glitters, then it is gold.”
 - b. “If you study linguistics and you study computer science, then you can get a job in natural language processing.”
 - c. “If don’t eat meat and you eat cheese, then you are a vegetarian.”
 - d. “If don’t eat meat and you don’t eat cheese, then you are vegan.”
2. For the following statement, write out the negation, converse, inverse, and contrapositive in English.

p = The animal has pointy ears, q = The animal is a cat,
 $p \rightarrow q$ = If the animal has pointy ears, then the animal is a cat.

 - a. Negation: $p \wedge \neg q$
 - b. Converse: $q \rightarrow p$
 - c. Inverse: $\neg p \rightarrow \neg q$
 - d. Contrapositive: $\neg q \rightarrow \neg p$
3. Complete truth tables for the following expressions, then explain the state of the propositional variables that cause the false case.
 - a. $p \rightarrow q$
 - b. $p \rightarrow (q \vee r)$
 - c. $p \rightarrow \neg(q \wedge r)$
 - d. $(p \wedge \neg q) \rightarrow r$

1.3: Implications (Conditional Propositions) - Answer key

1.
 - a. g is “it glitters”, o is “it is gold”.
 $g \rightarrow o$. (Your variables may be different.)
 - b. l is “you study linguistics”, c is “you study computer science”, n is “you get a job in natural language processing.”
 $(l \wedge c) \rightarrow n$
 - c. m is “you eat meat”, c is “you eat cheese”, v is “you are a vegetarian”.
 $(\neg m \wedge c) \rightarrow v$
 - d. m is “you eat meat”, c is “you eat cheese”, v is “you are a vegan”.
 $(\neg m \wedge \neg c) \rightarrow v$, or $\neg(m \vee c) \rightarrow v$
2.
 - a. Negation: $p \wedge \neg q$
The animal has pointy ears and it is not a cat.
 - b. Converse: $q \rightarrow p$
If the animal is a cat, then the animal has pointy ears.
 - c. Inverse: $\neg p \rightarrow \neg q$
If the animal does not have pointy ears, then the animal is not a cat.
 - d. Contrapositive: $\neg q \rightarrow \neg p$
If the animal is not a cat, then it does not have pointy ears.

3. a. $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

False when p is true and q is false.

b. $p \rightarrow (q \vee r)$

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

False when p is true and q and r are both false.

c. $p \rightarrow \neg(q \wedge r)$

p	q	r	$q \wedge r$	$\neg(q \wedge r)$	$p \rightarrow \neg(q \wedge r)$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

False when p is true and q and r are both true.

d. $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$(p \wedge \neg q)$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

False when p is true, q is false, and r is false.