Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own "dropbox" on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

1.5: Predicates and Quantifiers

1. $P(x)$ is the proposition, "x times 3 is even." State whether each of the following is *true* or *false*. The domain is $\mathbb{Z}$ - the set of integers.

   a. $P(1)$
   b. $P(2)$
   c. $P(5)$
   d. $P(10)$
   e. $P(2x)$

2. For each of the following, identify whether the statements are true for **all**, **some**, or **no** elements of the domain given.

   a. $O(n)$ is the predicate, "n is an odd number". The domain is $\mathbb{Z}$.
   b. $N(n)$ is the predicate, "n is a positive number". The domain is $\mathbb{N}$.
   c. $S(n)$ is the predicate, "n plus $n+1$ results in an odd number. The domain is $\mathbb{Z}$.
   d. $E(n)$ is the predicate, "2 times n plus 1 is an even number". The domain is $\mathbb{Z}$.

3. Translate the following into a symbolic statement. Define your predicate function and domain.

   a. "All college students are tired."
   b. "There is at least one college student who does not like pizza."
   c. "All college students who study will pass their class."
   d. "There is some college student who studied and did not pass their class."

4. Give some discrete (finite) domain that makes the following statement true for **all** elements of the domain.

   a. $A(x)$ is the predicate, "$x > 100$"

   b. $B(x)$ is the predicate, "$-2x > x$"

   c. $C(x)$ is the predicate, "$\sqrt{x} \in \mathbb{Z}$"

5. For each of the following statements, write the negation both symbolically and in English, and identify whether the **original is true** or the **negation is true**.

   a. $\forall x \in D, E(x) \wedge P(x)$
   Where $D = \{2, 4, 6, 8\}$, $E(x)$ is the predicate, "x is even", and $P(x)$ is the predicate, "$x > 0$".

   b. $\forall x \in E, O(x) \wedge P(x)$
   Where $E = \{1, 2, 3, 4\}$, $O(x)$ is the predicate, "x is odd", and $P(x)$ is the predicate, "$x > 0$".

   c. $\forall x \in \mathbb{Z}, N(x) \rightarrow \neg P(x)$
   Where $N(x)$ is the predicate, "$x < 0$". $P(x)$ is the predicate, "$x > 0$".

Predicates and Quantifiers - Answer key

1. a. $P(1) = 3 \cdot 1 = 3$; $P(1)$ is false.
   b. $P(2) = 3 \cdot 2 = 6$; $P(2)$ is true.
   c. $P(5) = 3 \cdot 5 = 15$; $P(5)$ is false.
   d. $P(10) = 3 \cdot 10 = 30$; $P(10)$ is true.
   e. $P(2x) = 3 \cdot 2x = 2(3x)$; $P(2x)$ is true. (2 times any integer is always even.)

2. a. True for some.
   b. True for some - the set of natural numbers includes 0, which is not positive.
   c. True for all - we will go into proofs on this later in this class.
   d. True for none - $2n + 1$ will always result in an odd number. We will learn the proof later.

3. a. $T(x)$ is the predicate, "x is tired". C is the set of all college students. (You might have used different letters in your definitions.)
   Statement: $\forall x \in C, T(x)$
   b. $P(x)$ is the predicate, "x does not like pizza". C is the set of all college students. $\exists x \in C, P(x)$
   (You might have also defined, $P(x)$ is "x likes pizza", which would give you $\exists x \in C, \neg P(x)$)
   c. $S(x)$ is the predicate, "x studies". $P(x)$ is the predicate, "x passes their class". C is the set of all college students.
   $\forall x \in C, S(x) \rightarrow P(x)$
   d. $S(x)$ is the predicate, "x studies". $P(x)$ is the predicate, "x passes their class". C is the set of all college students.
   $\exists x \in C, S(x) \wedge \neg P(x)$

4. a. Many answers. Example: Domain is $\{101, 102, 103\}$
   b. Many answers. Example: Domain is $\{-1, -2, -3\}$
   c. Many answers. Example: Domain is $\{4, 9, 16, 25\}$

5. a.

Original:	$\forall x \in D, E(x) \wedge P(x),\ D = \{2, 4, 6, 8\}$ "For all elements x in D, x is even and x is positive."
Negation:	$\neg(\forall x \in D, E(x) \wedge P(x))$ $\neg(\forall x \in D), \neg(E(x) \wedge P(x))$ $\exists x \in D, \neg E(x) \vee \neg P(x)$
English:	There exists some x in D such that x is not even or $x \leq 0$.
Result:	The original is true. The negation is false: No numbers are negative NOR are they odd.

b.

Original:	$\forall x \in E, O(x) \wedge P(x),\ E = \{1, 2, 3, 4\}$ "For all elements x in D, x is odd and x is positive."
Negation:	$\neg(\forall x \in E, O(x) \wedge P(x))$ $\neg(\forall x \in E), \neg(O(x) \wedge P(x))$ $\exists x \in D, \neg O(x) \vee \neg P(x)$
English:	There exists some x in D such that x is not odd or $x \leq 0$.
Result:	The original is false: Not all numbers are odd. The negation is true.

c.

Original:	$\forall x \in \mathbb{Z}, N(x) \rightarrow \neg P(x)$ "For all integers x, if x is negative then x is not positive."
Negation:	$\neg(\forall x \in \mathbb{Z}, N(x) \rightarrow \neg P(x))$ $\neg(\forall x \in \mathbb{Z}), \neg(N(x) \rightarrow \neg P(x))$ $\exists x \in \mathbb{Z}, N(x) \wedge \neg(\neg P(x))$ $\exists x \in \mathbb{Z}, N(x) \wedge P(x)$
English:	There exists some integer x such that $x < 0$ and $x > 0$.
Result:	The original is true. The negation is false: A number cannot be less than 0 and greater than 0 at the same time.

1.6: Predicates and Nested Quantifiers

1. Negate each of the following statements:

   a. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x)$

   b. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \wedge Q(x)$

   c. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x)$

   d. $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \rightarrow Q(x)$

   e. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x) \rightarrow R(x)$

   f. $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x) \rightarrow Q(x) \wedge R(x)$

Predicates and Nested Quantifiers - Answer key

1. Negate each of the following statements:

a.

Original: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x)$
Negation: $\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x))$
$\neg(\forall x \in \mathbb{Z}), \neg(\exists y \in \mathbb{Z}), \neg(P(x))$
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \neg P(x)$

b.

Original: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \wedge Q(x)$
Negation: $\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \wedge Q(x))$
$\neg(\forall x \in \mathbb{Z}), \neg(\exists y \in \mathbb{Z}), \neg(P(x) \wedge Q(x))$
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \neg P(x) \vee \neg Q(x)$

c.

Original: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x)$
Negation: $\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x))$
$\neg(\forall x \in \mathbb{Z}), \neg(\exists y \in \mathbb{Z}), \neg(P(x) \vee Q(x))$
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \neg P(x) \wedge \neg Q(x)$

d.

Original: $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \rightarrow Q(x)$
Negation: $\neg(\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \rightarrow Q(x))$
$\neg(\exists x \in \mathbb{Z}), \neg(\exists y \in \mathbb{Z}), \neg(P(x) \rightarrow Q(x))$
$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x) \wedge \neg Q(x)$

e.

Original: $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x) \rightarrow R(x)$
Negation: $\neg(\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \vee Q(x) \rightarrow R(x))$
$\neg(\forall x \in \mathbb{Z}), \neg(\exists y \in \mathbb{Z}), \neg(P(x) \vee Q(x) \rightarrow R(x))$
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x) \vee Q(x) \wedge \neg R(x)$

f.

Original: $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x) \rightarrow Q(x) \wedge R(x)$
Negation: $\neg(\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x) \rightarrow Q(x) \wedge R(x))$
$\neg(\forall x \in \mathbb{Z}), \neg(\forall y \in \mathbb{Z}), \neg(P(x) \rightarrow Q(x) \wedge R(x))$
$\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \wedge \neg(Q(x) \wedge R(x))$
$\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x) \wedge (\neg Q(x) \vee \neg R(x))$