Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in <u>one</u> copy of the exercise per group**.

Names:

Number Theory: Divisors

Quotient and Divisors

Let n and d be integers where $d \neq 0$. We say that d divides n if there exists some integer q satisfying

$$n = d \cdot q$$

We call q the **quotient** and d a **divisor** (or factor) of n.

If d divides n, we write $d|n$. If not, we write $d \nmid n$. [a]

[a] From Discrete Mathematics by Johnsonbaugh, p 223

Question 1
Mark each pairs of numbers as $d|n$ if d divides n, or $d \nmid n$ if not.

a. 2 _____ 4

b. 2 _____ 5

c. 7 _____ 49

d. k _____ $km + kn$

e. $n + 2$ _____ $n^2 + 5n + 6$

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Prime and Composite Numbers

An integer greater than 1 whose only positive divisors are itself and 1 is called **prime**. An integer that is greater than 1 and not prime is called **composite**. [a]

[a] From Discrete Mathematics by Johnsonbaugh, p 224

Fundamental Theorem of Arithmetic

Any integer greater than 1 can be written as a product of primes. [a]

[a] From Discrete Mathematics by Johnsonbaugh, p 227

Common Divisors and Multiples

Let m and n be integers with both m and n zero. A **common divisor** of m and n is an integer that divides both m and n.

The **Greatest Common Divisor** of two integers m and n (not both zero) is the largest positive integer that divides both m and n. The greatest common divisor can be written $gcd(m, n)$. [a]

Example:

The positive divisors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.
The positive divisors of 105 are: 1, 3, 5, 7, 15, 21, 35, 105.
The positive common divisors of 30 and 105 are: 1, 3, 5, and 15.
The greatest common divisor, $gcd(m, n)$ is 15.

Let m and n be positive integers. A **Least Common Multiple** of m and n is an integer that is divisible by both m and n that is the smallest positive common multiple of both. It can be written $lcm(m, n)$. [b]

Example:

The least common multiple of 30 and 105, $lcm(30, 105)$, is 210 because 210 is divisible by both 30 and 105.

[a] From Discrete Mathematics by Johnsonbaugh, p 228
[b] From Discrete Mathematics by Johnsonbaugh, p 230

Question 2
Find the greatest common divisor of each pair of integers: [1]

a. 0 and 17 b. 110 and 273 c. 20 and 40 d. 13 and 13^2

Question 3
Find the least common multiple of each pair of integers:

a. 3 and 4 b. 3 and 5 c. 4 and 10

[1] From Discrete Mathematics by Johnsonbaugh, p 233