Instructions: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in <u>one</u> copy of the exercise per group**.

Names:

Number Theory: Representations of Integers

Digits

For the decimal number $2,368$, we can write this as its individual digits:

Thousands (10^3)	**Hundreds** (10^2)	**Tens** (10^1)	**Ones** (10^0)
2	3	6	8

And then we can build out $2,368$ as the mathematical equation:

$$2 \cdot 10^3 + 3 \cdot 10^2 + 6 \cdot 10^1 + 8 \cdot 10^0$$

Likewise, for the binary number 0101 1001, we can write it as:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	1	0	0	1

And into the equation:

$$1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^0$$

Hexadecimal Digits

Hexadecimal numbers go from 0 to 15, but the two-digit numbers (10 through 15) are usually represented with letters so that you can write out a hex number with single characters.

A = 10 B = 11 C = 12 D = 13 E = 14 F = 15

Question 1

Expand each of the following numbers as a mathematical equation. Make sure to pay attention to the *base* value.

a. Write out the equation for $(19)_{10}$

10^1	10^0

b. Write out the equation for $(0010\ 1101)_2$ and calculate the decimal equivalent by expanding the equation and calculating the result.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

c. Write out the equation for $(FA6)_{16}$ and calculate the decimal equivalent by expanding the equation and calculating the result.

16^2	16^1	16^0

Converting between bases

Algorithm for converting a decimal number to base b:

1. Input a natural number n
2. While $n > 0$, do the following:
   (a) Divide n by b and get a quotient q and remainder r.
   (b) Write r as the next (right-to-left) digit.
   (c) Replace the value of n with q, and repeat.

Question 2
Convert the following between bases:

a. Convert $(35)_{10}$ to binary (base-2) $n = 35$, $b = 2$

b. Convert $(125)_{10}$ to binary (base-2) $n = 125$, $b = 2$

Hexadecimal to Binary

Often in computers, we write binary strings as hexadecimal to save space and make it easier to read.

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

Example: Convert 11001 from binary to hexadecimal

1. Write out in chunks of four. Add leading 0's to the left side.
   0001 1001
2. Swap out each "nibble" with hexadecimal
   0001 = 1 1001 = 9

So, $(0001\ 1001)_2 = (19)_{16}$

Example: Convert DAD from hexadecimal to binary

1. Convert each digit back to binary.
   D = 1101 A = 1010 D = 1101

So, $(\text{DAD})_{16} = (1101\ 1010\ 1101)_2$

Question 3

Do the following conversions

a. Convert $(1F0B)_{16}$ to binary:

b. Convert $(0100\ 0110)_2$ to hexadecimal:

Conversion cheat sheet

1. Converting from base b to base 10: Expand the number in base b to its digit-spaces and create an equation.

Example: Convert $(121)_3$ to base 10

3^2	3^1	3^0
1	2	1

$$= 1 \cdot 3^2 + 2 \cdot 3^1 + 1 \cdot 3^0$$

$$= 9 + 6 + 1$$

$$= 16$$

2. Converting between Binary and Hexadecimal: For converting between base-2 and base-16 (ONLY) you can use this table to swap out 4 bits at a time:

Hex	0	1	2	3	4	5	6	7
Binary	0000	0001	0010	0011	0100	0101	0110	0111
Hex	8	9	A	B	C	D	E	F
Binary	1000	1001	1010	1011	1100	1101	1110	1111

Example: Convert $(ABC)_{16}$ to binary

$$A = 1010 \qquad B = 1011 \qquad C = 1100$$

$$(ABC)_{16} = (\ \texttt{1010 1011 1100}\ )_2$$

Example: Convert $(\texttt{1111 1010 0010})_2$ to hexadecimal

$$1111 = F \qquad 1010 = A \qquad 0010 = 2$$

$$(\ \texttt{1111 1010 0010}\ )_2 = (FA2)_{16}$$

3. Convert decimal to binary: Use the algorithm!

Question 4
Convert each of the following using the appropriate methods.

a. $(10)_{10}$ to base-2.

b. $(67)_{10}$ to base-2.

c. (`1111 1010 1100 1110`)$_2$ to base-16.

d. (`1111 1010 1100 1110`)$_2$ to base-10.

e. $(1EE7)_{16}$ to base-2.

f. $(1EE7)_{16}$ to base-10.