

**Instructions:** Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own “dropbox” on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

## Number Theory: The Euclidean Algorithm

Johnsonbaugh: Chapter 5.2

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### The Euclidean Algorithm <sup>a</sup>

Input:  $a$  and  $b$  (nonnegative integers, not both zero).

Output: The Greatest Common Divisor of  $a$  and  $b$ .

```
gcd( a, b ) {  
    if ( a < b )  
        swap ( a, b );  
  
    while ( b != 0 ) {  
        r = a mod b;  
        a = b;  
        b = r;  
    }  
  
    return a;  
}
```

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<sup>a</sup>From Discrete Mathematics by Johnsonbaugh, p 249

1. Use the Euclidean Algorithm to find the greatest common divisor for each pair of numbers:
  - a. 60, 90
  - b. 315, 825
  - c. 2091, 4807

## Answer key

1. Convert each of the following from base-2 to base-16.

a. `gcd( 60, 90 )...`  
`a is greater than b, swap!`  
`while b != 0... (b = 60)`

Loop  
`a = 90 b = 60`  
`r = a % b (r = 30)`  
`a = b (a = 60)`  
`b = r (b = 30)`

Loop  
`a = 60 b = 30`  
`r = a % b (r = 0)`  
`a = b (a = 30)`  
`b = r (b = 0)`  
Return a (a = 30)  
Result: 30

```
b. gcd( 315, 825 )...  
  a is greater than b, swap!  
  while b != 0... (b = 315)
```

```
Loop  
a = 825  b = 315  
r = a % b (r = 195)  
a = b (a = 315)  
b = r (b = 195)
```

```
Loop  
a = 315  b = 195  
r = a % b (r = 120)  
a = b (a = 195)  
b = r (b = 120)
```

```
Loop  
a = 195  b = 120  
r = a % b (r = 75)  
a = b (a = 120)  
b = r (b = 75)
```

```
Loop  
a = 120  b = 75  
r = a % b (r = 45)  
a = b (a = 75)  
b = r (b = 45)
```

```
Loop  
a = 75  b = 45  
r = a % b (r = 30)  
a = b (a = 45)  
b = r (b = 30)
```

```
Loop  
a = 45  b = 30  
r = a % b (r = 15)  
a = b (a = 30)  
b = r (b = 15)
```

```
Loop  
a = 30  b = 15  
r = a % b (r = 0)  
a = b (a = 15)  
b = r (b = 0)  
Return a (a = 15)
```

```
Result: 15
```

```
c. gcd( 2091, 4807 )...  
  a is greater than b, swap!  
  while b != 0... (b = 2091)
```

```
Loop  
a = 4807  b = 2091  
r = a % b (r = 625)  
a = b (a = 2091)  
b = r (b = 625)
```

```
Loop  
a = 2091  b = 625  
r = a % b (r = 216)  
a = b (a = 625)  
b = r (b = 216)
```

```
Loop  
a = 625  b = 216  
r = a % b (r = 193)  
a = b (a = 216)  
b = r (b = 193)
```

```
Loop  
a = 216  b = 193  
r = a % b (r = 23)  
a = b (a = 193)  
b = r (b = 23)
```

```
Loop  
a = 193  b = 23  
r = a % b (r = 9)  
a = b (a = 23)  
b = r (b = 9)
```

```
Loop  
a = 23  b = 9  
r = a % b (r = 5)  
a = b (a = 9)  
b = r (b = 5)
```

```
Loop  
a = 9  b = 5  
r = a % b (r = 4)  
a = b (a = 5)  
b = r (b = 4)
```

```
Loop  
a = 5  b = 4  
r = a % b (r = 1)
```

## Discrete Structures I: Number Theory: Divisors

Textbooks: Johnsonbaugh: Chapter 5.2

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```
a = b (a = 4)
b = r (b = 1)
```

```
Loop
```

```
a = 4 b = 1
r = a % b (r = 0)
a = b (a = 1)
b = r (b = 0)
Return a (a = 1)
Result: 1
```