

Name: _____

Chapter 6 Exam

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Grading Scheme

#	Question	Weight	Points Received
1	Cards	20%	
2	Cards	20%	
3	Dice	20%	
4	Expected Value	20%	
5	Bernoulli Probability	20%	
6	Extra Credit 1	2%	
7	Extra Credit 2	2%	

Cheat sheet

Combinatorics - Structures:

	Repeats allowed?	Order matters?	Formula
Permutation <i>n</i> items to choose from Select <i>r</i> items	no	yes	$P(n, r) = \frac{n!}{(n-r)!}$
Set <i>n</i> items to choose from Select <i>r</i> items	no	no	$C(n, r) = \frac{n!}{r!(n-r)!}$
Ordered list <i>n</i> items to choose from Select <i>r</i> items	yes	yes	n^r
Unordered list <i>n</i> different types Select <i>r</i> items	yes	no	$C(n + r - 1, n)$
Unordered list for Binary Strings <i>r</i> 1's <i>n - r</i> 0's	yes	no	$C(n, r)$

The Sample Set S is the set of all possible outcomes, without restriction. $n(S)$ is the size of this set.

The Event E is the set of outcomes that match some event that we are concerned with, and $n(E)$ is the size of this set.

The probability of Event E occurring is given by $Prob(E) = \frac{n(E)}{n(S)}$

Two events are Disjoint (aka mutually exclusive) if they cannot occur simultaneously. This is handy for the sum rule

Two events are Independent if the occurrence of one event is not influenced by the occurrence (or non-occurrence) of the other event. This is handy for the product rule.

Cheat sheet

The General Sum Rule If E_1 and E_2 are any events in a given experiment, then the probability that E_1 OR E_2 occurs is given by:

$$Prob(E_1 \text{ or } E_2) = Prob(E_1) + Prob(E_2) - Prob(E_1 \text{ and } E_2)$$

If E_1 and E_2 are disjoint, then $Prob(E_1 \text{ and } E_2)$ is 0.

The General Product Rule If E_1 and E_2 are any events in a given experiment, then the probability that both E_1 AND E_2 occur is given by

$$Prob(E_1 \text{ and } E_2) = Prob(E_2) \cdot Prob(E_1|E_2)$$

...which is also equivalent to...

$$Prob(E_1 \text{ and } E_2) = Prob(E_2) \cdot Prob(E_2|E_1)$$

The probability of E_1 given E_2 Given events E_1 and E_2 for some experiment, we define the probability of E_1 happening given that E_2 occurred as by $Prob(E_1|E_2)$.

Probability in a Bernoulli Trial For a Bernoulli trial, we run the trial n times. We're looking for some success criteria to occur, and we want it to occur exactly k times. The probability of the success occurring is p . Given these variables, you can calculate the probability of the success occurring k times in n trials with:

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$$

Expected (average) value For a given experiment with a random number X , where the values that X could be come from the set $\{x_1, x_2, \dots, x_n\}$, the expected (average) value of X is:

$$E[X] = x_1 \cdot Prob(X = x_1) + \dots + x_n \cdot Prob(X = x_n)$$

Expected value in a Bernoulli trial Given a trial performed n times and the probability of success being p , the expected value in a Bernoulli trial is:

$$E[X] = n \cdot p$$

Question 1: Cards part 1

□ 0 □ 1 □ 2 □ 3 □ 4

Given a standard deck of 52 cards, we're going to draw cards without replacing them back in the deck. Answer the questions for the following scenarios.

A	2	3	4	5	6	7	8	9	10	J	Q	K
♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣
A	2	3	4	5	6	7	8	9	10	J	Q	K
♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠
A	2	3	4	5	6	7	8	9	10	J	Q	K
◇	◇	◇	◇	◇	◇	◇	◇	◇	◇	◇	◇	◇
A	2	3	4	5	6	7	8	9	10	J	Q	K
♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥

For the first experiment, you're drawing one card from the deck.

- The sample size, $n(S)$, is:
- What is the amount of events, $n(E_1)$, for the result of getting a value of 6:
- What is the amount of events, $n(E_2)$, for the result of getting a suit of diamonds:
- What is the amount of events, $n(E_3)$, for the result of getting a 6 of diamonds:
- Probability of getting a 6, $Prob(E_1)$:
- Probability of getting a diamond, $Prob(E_2)$:
- Probability of getting a 6 OR getting a diamond card, $Prob(E_1 \text{ and } E_2)$:

Question 2: Cards part 2

0 1 2 3 4

For the second experiment, you're drawing two cards from a deck.

a. What is the probability of getting an Ace as your first card, $Prob(E_{A1})$?

b. What is the probability of getting a Queen as your second card, given that you got an Ace as your first card? $Prob(E_{Q2}|E_{A1})$?

c. What is the probability of getting an Ace as your second card, given that you got an Ace as your first card? $Prob(E_{A2}|E_{A1})$?

Question 3: Dice roll

0 1 2 3 4

In an experiment, we are rolling two dice. All outcomes are:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Find the probability of the following.

- a. Getting an even number on the first die.

- b. Getting an even number exactly one die (not both).

- c. Getting at least one even number.

- d. The second die having a lower value than the first die.

Question 4: Bernoulli game

0 1 2 3 4

At a carnival game, you can play a coin flip game to earn money. In the game, you flip a coin 3 times. Every time you get a Heads, you get +\$5. Fill out the table below. The probability of a Bernoulli trial is $C(n, k) \cdot p^k \cdot (1-p)^{n-k}$.

- Probability of getting \$0:

- Probability of getting \$5:

- Probability of getting \$10:

- Probability of getting \$15:

Outcome	# Heads	Prize value x_i	$Prob(E_i)$
1	0	X = \$0	
2	1	X = \$5	
3	2	X = \$10	
4	3	X = \$15	

Next, calculate the expected average value, $E[X]$, with

$$E[X] = x_1 \cdot Prob(X = x_1) + \dots + x_n \cdot Prob(X = x_n)$$

Question 5: Another Bernoulli trial

0 1 2 3 4

In a die roll experiment, we are rolling a die 10 times. A single success is getting a value of 1 or 2. In this trial, we want to find the probability of getting at least 8 successes. Find the probability.

Question 6: Extra Credit 1

0 1 2 3 4

A box contains 15 toys. 3 toys in the box are broken. Let's say a kid will take 5 toys from the box...

- a. How many ways are there for the kid to select 5 toys with no restrictions?

- b. How many ways will there be exactly 1 broken toy?

- c. How many ways will contain all the broken toys?

- d. How many ways will contain no broken toys?

Question 7: Extra Credit 2

0 1 2 3 4

In a classroom, there are 10 computer science students, 8 IT students, and 5 math students. A president, vice president, and secretary must be elected. How many ways are there to elect these positions, with the additional constraints:

- a. No restrictions - anyone can serve any role.
- b. No Computer Science students can be elected.
- c. At least one Computer Science student must be elected.
- d. Either the president or the vice president must be an IT student, and CS and Math will be chosen for the other roles.