

INTRODUCTION TO PROBABILITY

ABOUT

Building on top of Chapter 5, we will be calculating the probability of some event(s) here in Chapter 6. Combinatorics will be used to figure out the size of our sample spaces and event spaces.

In this chapter, we need to go over terms.

TOPICS

1. Terminology

2. Example Problems

TERMINOLOGY

1. TERMINOLOGY

When we are figuring out the probability of something occurring, we need two pieces of information:

- 1) The amount of ways that event can occur, and
- 2) The total amount of ways, without restrictions.

Notes

1. TERMINOLOGY

Let's say we're flipping a coin three times. The total ways we can flip a coin three times without restriction is 8.

The sample set – or, writing out all ways this can occur – is the set S:

$$S = \{ \text{HHH}, \text{HHT}, \text{HTH}, \text{HTT}, \text{THH}, \text{THT}, \text{TTH}, \text{TTT} \}$$

Notes

1. TERMINOLOGY

Let's say we're flipping a coin three times. The total ways we can flip a coin three times without restriction is 8.

$S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \}$

For our probability computations, we don't need the set S to be written out, we only need the amount of elements in the set – written as $n(S)$.

$$n(S) = 8$$

Notes

1. TERMINOLOGY

Let's say we're flipping a coin three times. The total ways we can flip a coin three times without restriction is 8.

$S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \}$

We can also recognize that this is an ordered list, where each coin toss has 2 outcomes ($n = 2$) and we are flipping the coin 3 times ($r = 3$).

$$n(S) = 2^3 = 8$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

1. TERMINOLOGY

$$S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \}$$

We will also have some experiment E . For example, let's say we want to find the probability that we get exactly one Tails after flipping a coin 3 times.

Notes

$n(S)$ is the amount of total outcomes (the sample space)

1. TERMINOLOGY

$$S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \}$$

We will also have some experiment E . For example, let's say we want to find the probability that we get exactly one Tails after flipping a coin 3 times.

The event space is all the outcomes where this is true.

$$E = \{ \text{HHT, HTH, THH} \}$$

And the amount of outcomes, $n(E)$, is 3.

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

1. TERMINOLOGY

$$S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \}$$

$$E = \{ \text{HHT, HTH, THH} \}$$

We can also find this with our combinatorics:

Selecting a place for the 1 T first: $C(3,1)$

Then selecting the other Heads: 1^2

$$n(E) = 3 * 1 = 3$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

1. TERMINOLOGY

With the size of the sample space, $n(S)$, and the size of the event space, $n(E)$, we can find the probability with:

$$\mathbf{Prob(E) = n(E) / n(S)}$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $Prob(E) = n(E) / n(S)$

EXAMPLE PROBLEMS

2. EXAMPLE PROBLEMS

When picking one card out of a deck, find the probability that **the card is an Ace**.

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $Prob(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When picking one card out of a deck, find the probability that **the card is an Ace**.

$$n(S) = 52$$

$$n(E) = 4$$

$$\text{Prob}(E) = 4/52 = 1/13$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $\text{Prob}(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When picking one card out of a deck, find the probability that **the card is a red Ace**.

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $Prob(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When picking one card out of a deck, find the probability that **the card is a red Ace.**

$$n(S) = 52$$

$$n(E) = 2$$

$$\text{Prob}(E) = 2/52 = 1/26$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $\text{Prob}(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When rolling two dice – one is red and one is green – find the probability that **the dice sum to 2.**

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $Prob(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When rolling two dice – one is red and one is green – find the probability that **the dice sum to 2.**

$$n(S) = 36 \quad (6 \times 6)$$

$$n(E) = 1 \quad (\text{both dice are 1's})$$

$$\text{Prob}(E) = 1/36$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $\text{Prob}(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When rolling two dice – one is red and one is green – find the probability that **you get exactly one 5.**

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $Prob(E) = n(E) / n(S)$

2. EXAMPLE PROBLEMS

When rolling two dice – one is red and one is green – find the probability that **you get exactly one 5.**

$$n(S) = 36 \quad (6 \times 6)$$

$$E = \{ (1,5), (2,5), (3,5), (4,5), (6,5), \\ (5,1), (5,2), (5,3), (5,4), (5,6) \}$$

$$n(E) = 10$$

$$\text{Prob}(E) = 10/36 = 5/18$$

Notes

$n(S)$ is the amount of total outcomes (the sample space)

$n(E)$ is the amount of outcomes that meet the event E

Probability of E occurring:
 $\text{Prob}(E) = n(E) / n(S)$

CONCLUSION

This is just the basics of probability. In 6.2 we will apply the Rule of Sums and the Rule of Products that we learned for Combinatorics, but use it with Probability.