

Name: _____

Chapter 6 EXAM C:

This exam covers concepts from Chapter 6. Questions will be labelled by the section from which they are based.

Make sure to answer the questions clearly and show your work to get full credit.

This exam is to be **solo effort**. Any reasonable instance of cheating will result in a 0% for those participating.

You can use a standard calculator for this exam, but not a graphing calculator.

Each question can receive between 0 and 4 points, and each question has a weight associated with it. The point value is used to compute the score for a question. For example, if a question is worth a weight of 5% and the student receives 3 points, then that question will count for 3.75% out of the full 5%.

0	1	2	3	4
Nothing written	Attempted, but incorrect	Partially correct; multiple errors	Mostly correct, one or two errors	Perfect; correct answer & notation

Grading breakdown:

Question	Weight	Points Received	Weighted Score
1	12%		
2	8%		
3	14%		
4	14%		
5	12%		
6	10%		
7	15%		
8	15%		
9	+4%		
10	+1%		

12% Question 1: Drawing a single card

□ 0 □ 1 □ 2 □ 3 □ 4

For an experiment where a single card is drawn from a standard deck of 52 cards, fill out the following table and then answer the questions.

Note: Ace is not a face card, and numbered cards are cards 2 through 10.

<i>Event</i>	$n(E)$	$n(S)$	$Prob(E)$
The card is a black card.			
The card has a spade suit.			
The card is a king of spades.			
The card has an odd number value ¹ .			
The card is a face card.			

- What is the probability that the card is either a Spade or a Club?
- What is the probability that the card has a value of Queen AND a suit of Spades?
- What is the probability that the card either has a value of Queen OR a suit of Spades?

8% Question 2: Drawing two cards 0 1 2 3 4

For an experiment where two cards are selected out of a single deck **with no replacement**, find the probability that...

a. The first card drawn is a King, and the second card drawn is an Ace.

b. Both cards have the same value.

14% Question 3: Rolling one die 0 1 2 3 4

In an experiment, a machine randomly gives you a number between 1 and 10 (inclusive). Each outcome is equally probable. What's the probability that you get a value of 6 or more?

14% Question 4: Probability of this given that 0 1 2 3 4

In an experiment, you are drawing two cards from a standard deck. Event E_1 is getting a heart card as the first card, and event E_2 is getting a heart card as the second card. What is the probability of E_2 occurring, given that E_1 occurred. In other words, what is $Prob(E_2|E_1)$?

12% Question 5: Rolling two dice 0 1 2 3 4

In an experiment, we are rolling two dice. All of the outcomes are:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Find the probability of the following.

- The probability of getting the same number for both the first and second dice.
- The probability of the second die value being greater than or equal to the first die value.

10% Question 6: Probability in a Bernoulli Trial 0 1 2 3 4

In a die roll experiment, we are rolling a die 5 times. A single success is getting a value of 1 or 2. In this trial, we want to find the probability of getting two successes out of the 5 trials. Find the probability.

15% Question 7: Probability in a Bernoulli Trial 0 1 2 3 4

In a die roll experiment, we are rolling a die 5 times. A single success is getting a value of 1 or 2. In this trial, we want to find the probability of getting at least two successes out of the 5 trials. Find the probability.

15% Question 8: Expected value

□ 0 □ 1 □ 2 □ 3 □ 4

In a game, you will flip a coin 3 times. If you get a Heads, then you win \$3. Find the probability for each outcome using the Bernoulli formula,

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$$

Outcome	# Heads	Prize value x_i	$Prob(E_i)$
1	0	$X = \$0$	
2	1	$X = \$3$	
3	2	$X = \$6$	
4	3	$X = \$9$	

Next, calculate the expected/average value $E[X]$ with

$$E[X] = x_1 Prob(X = x_1) + \dots + x_n Prob(X = x_n)$$

Cheat sheet

Cheat sheet

Disjoint events Two events are said to be **disjoint** (or *mutually exclusive*) if they cannot occur simultaneously.

Independent events Two events are said to be **independent** if the occurrence of one event is not influenced by the occurrence (or nonoccurrence) of the other event.

The General Sum Rule If E_1 and E_2 are any events in a given experiment, then the probability that E_1 or E_2 occurs is given by

$$Prob(E_1 \text{ or } E_2) = Prob(E_1) + Prob(E_2) - Prob(E_1 \text{ and } E_2)$$

If E_1 and E_2 are disjoint, then $E_1 \cap E_2 = \emptyset$, so $Prob(E_1 \text{ and } E_2) = 0$.

The General Product Rule If E_1 and E_2 are any events in a given experiment, then the probability that both E_1 and E_2 occur is given by
 $Prob(E_1 \text{ and } E_2) = Prob(E_2) \cdot Prob(E_1 | E_2)$
 $= Prob(E_1) \cdot Prob(E_2 | E_1)$

The probability of E_1 given E_2 Given events E_1 and E_2 for some experiment, we define the probability of E_1 given E_2 , denoted by $Prob(E_1 | E_2)$, as the probability that E_1 happens given that E_2 occurs. Note that if E_1 and E_2 are independent, then $Prob(E_1 | E_2) = Prob(E_1)$.

Cheat sheet

Probability Given an experiment with a sample space S of equally likely outcomes and an event E , the probability of the event occurring, written as $Prob(E)$, is

$$Prob(E) = \frac{n(E)}{n(S)}$$

Complement Given an event E ,

$$Prob(E) + Prob(\bar{E}) = 1$$

where \bar{E} is the complement of the event E .

Probability in a Bernoulli Trial For a Bernoulli trial, we run a trial n times. We're looking for some success to happen, and we want it to occur exactly k times. The probability of the success occurring is p . Given these, then you can calculate the probability of k successes occurring with:

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}$$

Expected (average) value For a given experiment, let X be a random variable whose possible values come from the set $\{x_1, \dots, x_n\}$. The expected value of X , denoted by $E[X]$, is the sum:

$$E[X] = x_1 Prob(X = x_1) + \dots + x_n Prob(X = x_n)$$

Expected value in a Bernoulli trial Given a trial performed n times and the probability of success being p , the expected value $E[X]$ is

$$E[X] = np$$