6.1 Exercise: In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. Work in a team of up to 4 people to complete this exercise. You can work simultaneously on the problems, or work separate and then check your answers with each other. You can take the exercise home, score will be based on the in-class quiz the following class period. **Work out problems on your own paper** - this document just has examples and questions.

6.1 Intro to Probability

6.1.1 Experiments, Outcomes, and Events

Vocabulary

For this chapter, we will be talking about experiments and their outcomes. For any given experiment, we will have a **sample space** of possible outcomes. This will be written as the set S.

Within an experiment, we want to see if some **event** occurs, and how often it does. In cases where the event occurs, we call it a **success**.

Definition Given an experiment with a sample space S of equally likely outcomes and an event E, the *probability of the event* (denoted by $Prob(E)$) is the ratio of the number of successful outcomes to the total number of outcomes: [a]

$$Prob(E) = \frac{n(E)}{n(S)}$$

(Recall that $n(S)$ is how we symbolically write, "the amount of elements of the set S".)

[a] From Discrete Mathematics by Ensley and Crawley

Question 1

Finish the following table to log all possible equally-likely outcomes for rolling a red six-sided die and a green six-sided die.

	G 1	**G 2**	**G 3**	**G 4**	**G 5**	**G 6**
R 1	(1, 1)	(1, 2)				
R 2	(2, 1)	(2, 2)				
R 3			(3, 3)			
R 4				(4, 4)		
R 5					(5, 5)	
R 6						(6, 6)

Using the definition above describe the following:

a. Both the red and green dice have the same values.

$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

b. The sum of both dice values is 4.

$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

c. **Exactly** one die has the value "4".

$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

d. **At least** one die has the value "4".

$n(E) =$ _____ $n(S) =$ _____ $Prob(E) =$ _____

Question 2
For a scenario where we roll two dice, fill out the table with all possible roll outcomes that give a sum from the minimum sum, 2, to the maximum sum, 12.

Dice sum	**Outcomes**	$n(E)$
2	*(1, 1)*	
3	*(2, 1), (1, 2)*	
4		
5		
6		
7		
8		
9		
10		
11		
12	*(6, 6)*	
$n(S)$		

a. How many total outcomes are there for rolling two dice?

b. What is the probability that rolling two dice will result in a sum of 12?

c. What is the probability that rolling two dice will result in a sum of 8?

d. What is the probability that rolling two dice will result in a sum greater than 7?

Question 3

Consider the experiment of drawing two cards from the top of a standard deck of 52 cards, and the event E of the two cards having the same value. [1]

a. If we're drawing two cards from a deck of 52, what is the **structure**? What is n and r? This will be the size of the sample space, $n(S)$.

b. Use combinatorics to figure out the amount of outcomes, $n(E)$, that result in E being successful. (i.e., how do calculate the amount of outcomes where two cards have the same values?)

Any card		Any card with same value
52	$\times$	3

$n(E) =$

c. Compute $Prob(E) = \frac{n(E)}{n(S)}$.

$Prob(E) = \frac{n(E)}{n(S)} =$

[1] From Discrete Mathematics by Ensley and Crawley

Question 4
Consider the experiment of tossing a coin five successive times, and the event E that the last two tosses have the same result.

(__ __ __ Heads Heads) OR (__ __ __ Tails Tails)

a. n = amount of outcomes =

   r = amount of trials =

   Structure is =
   (combination / permutation / ordered list / unordered list)

   $n(S)$ =

b. How many outcomes are there where the last two coins result in heads?

c. How many outcomes are there where the last two coints result in tails?

d. What rule do we use to get the total amount of outcomes that match our event E? (rule of sums / rule of products)

e. $n(E)$ =

f. $Prob(E) = \frac{n(E)}{n(S)} =$

6.1.2 The Complement of the Event

Proposition 1
Given an event E,

$$Prob(E) + Prob(\bar{E}) = 1$$

Where $\bar{E}$ is the complement of the event E.