Probability: Sum and Product Rule

Bernoulli Trials

Theorem 1
Given a simple experiment, called a **Bernoulli trial**, and an event that occurs with a probability p, if the trial is repeated independently n times, then the probability of having exactly k successes is [a]

$$C(n,k) \cdot p^k \cdot (1-p)^{n-k}$$

Example 1 What is the probability that in 10 successive rolls of a fair, six-sided die, we get exactly five results of 6?

Here, we have $n = 10$, $k = 5$, and $p = \frac{1}{6}$, so:

$$C(10,5) \cdot (\frac{1}{6})^5 \cdot (1 - \frac{1}{6})^{10-5}$$

$$\frac{10!}{5!(10-5)!} \cdot (\frac{1}{6})^5 \cdot (\frac{5}{6})^5$$

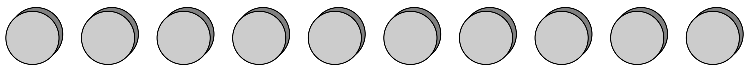
$$\frac{3628800}{14400} \cdot \frac{1}{7776} \cdot \frac{3125}{7776}$$

$$\approx 0.013$$

[a] From Discrete Math by Ensley and Crawley, page 460

Question 1

What is the probability of getting exactly 3 heads on 10 tosses of a fair coin?



n, the amount of trial repeats:

k, the amount of successes (heads):

p, the probability of success:

Use the formula of $C(n, k) \cdot p^k \cdot (1-p)^{n-k}$ to find the probability of getting 3 heads on 10 tosses of a fair coin.

By Rachel Singh, last updated November 1, 2018

Question 2
What is the probability that in seven rolls of a six-sided die, the result of 1 appears *at least* five times?

Hint
For this one, we will need to use the **rule of sums** to combine several outcomes: Getting 5 1's, 6 1's, OR 7 1's.

		repeats n	successes k	probability p
A	Getting five 1's	7	5	1/6
B	Getting six 1's	7		
C	Getting seven 1's	7		

Now, using the formula $C(n,k) \cdot p^k \cdot (1-p)^{n-k}$ three different times for case (A), (B), and (C).

(A) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

(B) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

(C) $C(n,k) \cdot p^k \cdot (1-p)^{n-k} =$

To find the probability of getting at least five 1's in seven rolls, add (A), (B), and (C) together. (Just write out the formula; don't solve.)

Prob(at least five 1's) =