Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own "dropbox" on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

2.1: Mathematical Systems, Direct Proofs, and Counterexamples

1. Prove each of the following with a direct proof:
   a. If n is **divisible by 6**, then n is **even**.
   b. If n is **even**, then $n + 1$ is **odd**.
   c. If n is a positive **odd** integer, then $n^2 + n$ is **even**.
2. Prove each of the following with a direct proof:[1]
   a. For all integers a and b, if a is **even** and b is **even**, then $a + b$ is **even**.
   b. For all integers c and d, if c is **odd** and d is **odd**, then $c + d$ is **even**.
   c. For all integers e and f, if e is **even** and f is **even**, then ef is **even**.
   d. For all integers g and h, if g is **odd** and h is **odd**, then gh is **odd**.
   e. For all integers i and j, if i is **even** and j is **odd**, then ij is **even**.
3. Prove the following with a direct proof:[2]
   For all integers n, if n is **even**, then the product of n and its successor is **even**.

[1] From Discrete Mathematics, 7th ed, Richard Johnsonbaugh, page 75
[2] From Discrete Mathematics, Ensley and Crawley, page 97

Mathematical Systems, Direct Proofs, and Counterexamples - Answer key

1. a. $n = 6k$, $\boxed{n = 2(3k)}$

   b. $n = 2k$, $n + 1 \equiv \boxed{2k + 1}$

   c. $n = 2k + 1$,
   $n^2 + n \equiv (2k + 1)^2 + (2k + 1)$
   $(2k + 1)^2 + (2k + 1) \quad = 4k^2 + 4k + 1 + 2k + 1 \quad = 4k^2 + 6k + 2$
   $\boxed{= 2(2k^2 + 3k + 1)}$

2. a. For all integers a and b, if a is **even** and b is **even**, then $a + b$ is **even**.
   1. Definition of an even number: $n = 2k$.
   2. $a = 2k$, $b = 2j$ (make sure to use different variables for each).
   3. Change $a+b$ to $2k+2j$ and rearrange into the definition of an even number to prove the statement.
   4. $2k + 2j = 2(k + j)$ $\boxed{2(k + j) \text{ is an even integer.}}$

   b. For all integers c and d, if c is **odd** and d is **odd**, then $c + d$ is **even**.
   1. The definition of an odd number: $n = 2k + 1$.
   2. $c = 2k + 1$, $d = 2j + 1$ (make sure to use different variables for each).
   3. Change $c + d$ to $2k + 1 + 2j + 1$ and rearrange into the definition of an even number to prove the statement.
   4. $2k+1+2j+1 = 2k+2j+2 = 2(k+j+1)$ $\boxed{2(k + j + 1) \text{ is an even integer.}}$

   c. For all integers e and f, if e is **even** and f is **even**, then ef is **even**.
   1. $e = 2k$, $f = 2j$
   2. $ef \equiv (2k)(2j)$
   3. $(2k)(2j) = 2(2kj)$ $\boxed{2(kj) \text{ is an even integer.}}$

   d. For all integers g and h, if g is **odd** and h is **odd**, then gh is **odd**.
   1. $g = 2k + 1$, $h = 2j + 1$
   2. $gh \equiv (2k + 1)(2j + 1)$
   3. $(2k + 1)(2j + 1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$
   $\boxed{2(2kj + k + j) + 1 \text{ is an odd integer.}}$

   e. For all integers i and j, if i is **even** and j is **odd**, then ij is **even**.
   1. $i = 2k$, $j = 2j + 1$
   2. $ij \equiv (2k)(2j + 1)$
   3. $(2k)(2j+1) = 4kj+2k = 2(2kj+k)$ $\boxed{2(2kj + k) \text{ is an even integer.}}$

3. For all integers n, if n is **even**, then the product of n and its successor is **even**.
   1. The problem here is $n(n + 1)$. n is even, so $n = 2k$.
   2. $n(n + 1) \equiv (2k)(2k + 1)$, $(2k)(2k + 1) = 4k^2 + 2k = 2(2k^2 + k)$
   $\boxed{2(2k^2 + k) \text{ is an even integer.}}$