Instructions: Work on homework assignments to further familiarize yourself with the topics in the class. The answers are provided for these problems. You can work with other students as desired. Turn in your work on canvas to be given a grade for completion (homework will not be checked for correctness; you need to verify this yourself.)

Upload each homework assignment to its own "dropbox" on Canvas.

This document is not formatted to be written on; do your homework on a separate sheet of paper.

2.2: More Methods of Proof

1. Prove each of the following with a proof by contradiction:
   a. For every $n \in \mathbb{Z}$, if n^2 is even, then n is even. [1]
   b. For every $n \in \mathbb{Z}$, if n^2 is odd, then n is odd. [2]
2. Prove each of the following with a proof by contrapositive:
   a. For every $n \in \mathbb{Z}$, if n^2 is even, then n is even.
   b. For every $n \in \mathbb{Z}$, if n^2 is odd, then n is odd. [3]

[1] Discrete Mathematics 7th ed, Johnsonbaugh, p 77
[2] Discrete Mathematics 7th ed, Johnsonbaugh, p 86
[3] Discrete Mathematics 7th ed, Johnsonbaugh, p 86

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More Methods of Proof - Answer key

1. a. Original: If n^2 is even, then n is even.
Negation: n^2 is even and n is not even.
$n^2 = n \cdot n$
Make alias variables for both sides: $n^2 = 2k$ $n = 2j + 1$.
$n^2 = n \cdot n$ rewrite as $2k = (2j + 1)(2j + 1)$
Simplify and move constants to one side...
$2k = 4j^2 + 4j + 1$ ... $2k - 4j^2 - 4j = 1$ ...
$2(k - 2j^2 - 2j) = 1$ ... divide both sides by 2
$\boxed{k - 2j^2 - 2j = \frac{1}{2}}$
We've discovered a contradiction - the solution cannot have a rational number, since we are dealing with integers!

   b. Original: If n^2 is odd, then n is odd.
Negation: n^2 is odd and n is not odd.
$n^2 = n \cdot n$
Make alias variables for both sides: $n^2 = 2k + 1$ $n = 2j$.
$n^2 = n \cdot n$ rewrite as $2k + 1 = (2j)(2j)$
Simplify and move constants to one side...
$2k + 1 = 4j^2$ ... $1 = 4j^2 - 2k$
$1 = 2(2j^2 - k)$ ... Divide both sides by 2
$\boxed{\frac{1}{2} = 2j^2 - k}$
We've discovered a contradiction!

2. a. Original: If n^2 is even, then n is even.
Contrapositive: If n is not even, then n^2 is not even.
$n = 2k + 1$, then find n^2 through this.
$n^2 \equiv (2k + 1)^2$ $= 4k^2 + 4k + 1$ $\boxed{= 2(2k^2 + 2k) + 1}$

   b. Original: If n^2 is odd, then n is odd.
Contrapositive: If n is not odd, then n^2 is not odd.
$n = 2k$, then find n^2 through this.
$n^2 \equiv (2k)^2$ $= 4k^2$ $\boxed{= 2(2k^2)}$

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