**Instructions:** In-class exercises are meant to introduce you to a new topic and provide some practice with the new topic. **Work in a team of up to 4 people to complete this exercise.** You can work simultaneously on the problems, or work separate and then check your answers with each other. **Turn in one copy of the exercise per group**.

**Names:**

# Sets: Set Operations

## Partitions

**Partitions**

The Partition of a set, usually denoted by $S$, is a **set of subsets** that, when combined together, form the **original set**.

**Definition:** For a set $A$, a partition of $A$ is some set $S = \{S_1, S_2, S_3, ...\}$ of subsets of $A$, such that:

1. For all $i$, $S_i \neq \emptyset$ - that is, each *part* is non-empty.
2. For all $i$ and $j$, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$ - that is, different *parts* have nothing in common.
3. $S_1 \cup S_2 \cup S_3 \cup ... = A$ - that is, every element in $A$ is contained in some *part*.

**Clarification:** The partition is known as $S$. Each element of $S$, such as $S_i$, is known as a **part**. A Part is a set as well. No parts are empty sets, and all parts must have some elements that come from $A$. An element of $A$ cannot be repeated across multiple Parts, and all elements of $A$ must be represented in the entire partition $S$.


By Rachel Singh, last updated October 1, 2018

**Partition Example**

Let's say we have a set, $A = \{1, 2, 3, 4\}$. We could form multiple partitions, such as:

- Partition 1: $\{\{1\}, \{2\}, \{3\}, \{4\}\}$
- Partition 2: $\{\{1, 2\}, \{3, 4\}\}$
- Partition 3: $\{\{1, 2, 3\}, \{4\}\}$
- Partition 4: $\{\{1, 2, 3, 4\}\}$

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### Question 1

Write out all possible partitions of $A = \{1, 2\}$. There should be 2. Note that the order of the elements of the set does not matter.

1.

2.

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### Question 2

Write out all possible partitions of $B = \{1, 2, 3\}$. There should be 5.

1.

2.

3.

4.

5.

## Power sets

**Power Sets**
The Power Set of $A$ is defined as $\wp(A) = \{S : S \subseteq A\}$. In other words, the Power Set is the **set of all possible subsets that you could build from $A$, INCLUDING the empty set.**

**Example 1:** Find the Power Set of $\{A\}$.

$$\wp(\{A\}) = \{\emptyset, \{A\}\}$$

**Example 2:** Find the Power Set of $\{A, B\}$.

$$\wp(\{A, B\}) = \{\emptyset, \{A\}, \{B\}, \{A, B\}\}$$

**Example 3:** Find the Power Set of $\{A, B, C\}$.

$$\wp(\{A, B, C\}) = \{\emptyset, \{A\}, \{B\}, \{C\}, \{A, B\}, \{B, C\}, \{A, C\}, \{A, B, C\}\}$$

**Example 4:** Find the Power Set of $\{A, B, C, D\}$.
$\wp(\{A, B, C, D\}) = \{$
$\emptyset,$
$\{A\}, \{B\}, \{C\}, \{D\},$
$\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\},$
$\{A, B, C\}, \{A, B, D\}, \{A, C, D\}, \{B, C, D\},$
$\{A, B, C, D\}$
$\}$
(Phew!)

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**Question 3**
Find the Power Set for the following:

a. $\wp(\{1, 2\}) =$

b. $\wp(\{3, 4\}) =$

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**Question 4**
Find the Power Set for $\wp(\{1, 2, 3\})$.

## Cartesian products

**Cartesian Products**
We can compute the Cartesian Product of two sets, such as $A$ and $B$. The result will be a set of **ordered pairs**, such as $(a, b)$, combining the elements of $A$ and $B$ together.

**Example:** For $A = \{1, 2\}$ and $B = \{4, 5, 6\}$, find $A \times B$.

| | $B_1 = 4$ | $B_2 = 5$ | $B_3 = 6$ |
|---|---|---|---|
| $A_1 = 1$ | $(1, 4)$ | $(1, 5)$ | $(1, 6)$ |
| $A_2 = 2$ | $(2, 4)$ | $(2, 5)$ | $(2, 6)$ |

So the final result is:

$$A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6)\}$$

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**Question 5**
Find the Cartesian Product of $C \times S$ given $C = \{red, green\}$ and $S = \{shirt\}$

| | $C_1 = red$ | $C_2 = green$ |
|---|---|---|
| $S_1 = shirt$ | | |

$C \times S =$

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**Question 6**
Given the sets find the following Cartesian Products.

$$N = \{1, 2\} \qquad L = \{a, b\}$$

a. $N \times L$

| | $N_1 = 1$ | $N_2 = 2$ |
|---|---|---|
| $L_1 = a$ | | |
| $L_2 = b$ | | |

$N \times L =$

b. $L \times N$

| | $L_1 = a$ | $L_2 = b$ |
|---|---|---|
| $N_1 = 1$ | | |
| $N_2 = 2$ | | |

$L \times N =$

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**Question 7**
Given the sets find the following Cartesian Products.

$$C = \{x, y, z\} \qquad Z = \{5, 10\}$$

a. $C \times Z$

| | $C_1 = x$ | $C_2 = y$ | $C_3 = z$ |
|---|---|---|---|
| $Z_1 = 5$ | | | |
| $Z_2 = 10$ | | | |

$C \times Z =$

b. $Z \times C$

| | $Z_1 = 5$ | $Z_2 = 10$ |
|---|---|---|
| $C_1 = x$ | | |
| $C_2 = y$ | | |
| $C_2 = z$ | | |

$Z \times C =$

## Common pitfalls

**Common pitfalls**

Students frequently mix up Power Sets, Partitions, and Cartesian Products. Make sure that you understand the difference between each:

- Partition of $X$: A set of sets (parts), where each set contains some elements of the original set $X$. No sets can contain the same items, and no set can be empty. All elements of $X$ must be represented in the partition.
- Power Set of $X$: A set of sets that gives you all possible subsets of the original set $X$, *including* the empty set.
- Cartesian Product of $X$ and $Y$: A set of ordered pairs that combines all elements of $X$ with all elements of $Y$.

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**Question 8**

Find the Power Set for the following.

$\wp(\{red, green, blue\}) =$

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**Question 9**
For the set $A = \{1, 2, 3, 4, 5, 6\}$, build partitions that meet the following criteria:

a. Find a partition where each part has the same size.

b. Find a partition where no two parts have the same size.

c. Find a partition that has as many parts as possible.

d. Find the partition that has as few parts as possible.

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**Question 10**
Compute the following Cartesian Products given the following sets.

$$E = \{2, 4\} \qquad O = \{1, 3, 5\} \qquad A = \{a, e, i, o\}$$

a. $E \times O =$

b. $A \times E =$

c. $E^2$ (hint: $E \times E$) $=$