Sets

Written by Rachel Singh, last updated Oct 1, 2018

About

Sets are a way we can organize data in a group under a common name. In programming, you might have worked with Arrays or Lists – Sets are similar, but also have their differences.

Topics

1. What is a set?
2. Subsets and equivalent sets
3. Set operations
4. Venn diagrams
5. Complex operations
6. DeMorgan’s Laws

1. What is a set?

1. What is a set?

A set is a grouping of items together. They can be numbers, but also other classifications like a list of classes or a list of people.

Set names are usually capital letters, such as

$$A = \{ 1, 2, 3, 4 \}$$

Notes

1. What is a set?

$A = \{1, 2, 3, 4\}$

An element of a set is one of the items within that list. In the set above, 1, 2, 3, and 4 are all elements.

The order of items in a set **does not matter**, so these two sets are considered equivalent:

$A = \{1, 2, 3, 4\}$ $B = \{4, 2, 3, 1\}$

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

1. What is a set?

Having duplicates also does not matter – the following sets are considered equivalent.

$A = \{ 1, 2, 3, 4 \}$ $C = \{ 1, 1, 2, 3, 4, 3 \}$

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

1. What is a set?

A set can contain a discrete (fininte) amount of elements, like A:

$$A = \{ 1, 2, 3, 4 \}$$

But it can also contain an infinite amount of elements, such as the set of all integers:

$$\mathbb{Z}$$

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

1. What is a set?

The following sets are commonly used, so they have their own symbols:

$\mathbb{Z}$	**Integers** Whole numbers including positive, negative, and 0.
$\mathbb{R}$	**Real numbers**
$\mathbb{N}$	**Natural numbers** “Counting numbers” - positive integers.
$\mathbb{Q}$	**Rational numbers** Any number that can be represented fractionally.

Notes

A **set** is a list of elements.

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Duplicates do not matter in a set.

1. What is a set?

We can make these further specific by specifying whether it will be

Non-negative: $\mathbb{Z}^{\geq 0}$ or $\mathbb{Z}^{nonneg}$

"The set of all non-negative integers"

Positives only: $\mathbb{Z}^{+}$

"The set of all positive integers"

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

$\mathbb{Q}$ Rational
$\mathbb{N}$ Natural
$\mathbb{R}$ Real
$\mathbb{Z}$ Integers

1. What is a set?

We can also define our own sets with infinite elements, but we need a special way to write it. That's where set notation comes in:

A = { x | x is an even integer }

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

$\mathbb{Q}$	Rational
$\mathbb{N}$	Natural
$\mathbb{R}$	Real
$\mathbb{Z}$	Integers

1. What is a set?

A couple of other sets we will be using is the universal set and the empty set.

U, the universal set, will contain all elements of all sets in a given problem.

Ø, the empty set, is a set that contains no elements.

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

$\mathbb{Q}$ Rational
$\mathbb{N}$ Natural
$\mathbb{R}$ Real
$\mathbb{Z}$ Integers

1. What is a set?

Finally, we can specify *how many elements a set has* by writing:

$$|X|$$

or, in some books:

$$n(X)$$

This is the <u>cardinality</u> of set X.

Notes

A **set** is a list of elements.

The **order** does not matter in a set.

Duplicates do not matter in a set.

$\mathbb{Q}$ Rational
$\mathbb{N}$ Natural
$\mathbb{R}$ Real
$\mathbb{Z}$ Integers

The **cardinality** of X is the # of elements in X, written |X|.

2. Subsets and Equivalent Sets

2. Subsets and Equivalent Sets

We can compare whether two sets or equivalent, or if one set is a subset of another.

= shows equivalence

⊆ shows “is a subset or equivalent to”
- Can be a subset or equivalent to.

⊂ shows “is a proper subset of”
- Cannot also be equivalent.

⊄ shows “is NOT a subset of”

Notes

The **cardinality** of X is the # of elements in X, written |X|.

= equivalence

⊆ “is a subset OR equivalent to”

⊂ “is a proper subset of”

⊄ “NOT a subset of”

2. Subsets and Equivalent Sets

For two sets to be equivalent, all elements from one set must also be present in the other set, and no additional items.

A = { 1, 2, 3, 4 }

C = { 1, 1, 2, 3, 4, 3 }

D = { 1, 2, 3, 4, 5 }

A = C
A is equivalent to C

A ⊂ D or A ⊆ D
A is a subset of D

for every x in A, if x is also in B, AND…
for every x in B, if x is also in A, THEN…
A and B are equivalent.

Notes

The **cardinality** of X is the # of elements in X, written |X|.

= equivalence

⊆ "is a subset OR equivalent to"

⊂ "is a proper subset of"

⊄ "NOT a subset of"

2. Subsets and Equivalent Sets

If A = B, then B = A.

If A ⊆ B, then it is *possible* that B ⊆ A, but it is not guaranteed.

If A ⊂ B, then it is *NOT POSSIBLE* that B ⊂ A.

Notes

The **cardinality** of X is the # of elements in X, written |X|.

= equivalence

⊆ "is a subset OR equivalent to"

⊂ "is a proper subset of"

⊄ "NOT a subset of"

2. Subsets and Equivalent Sets

A = { 1, 2, 3, 4 } ***D = { 1, 2, 3, 4, 5 }***

A ⊂ D **but** **D ⊄ A**

A is a subset of D,
but D is not a subset of A.

Notes

The **cardinality** of X is the # of elements in X, written |X|.

= equivalence

⊆ “is a subset OR equivalent to”

⊂ “is a proper subset of”

⊄ “NOT a subset of”

2. Subsets and Equivalent Sets

We also use the symbol ∈ to say that some **element** exists in some **set**…

$$A = \{1, 2, 3, 4\}$$

$$1 \in A$$

$$2 \in A$$

x ∈ A can be read, “x in A”,
“x exists in A”, “x is an element of A”.

Notes

The **cardinality** of X is the # of elements in X, written |X|.

= equivalence

⊆ “is a subset OR equivalent to”

⊂ “is a proper subset of”

⊄ “NOT a subset of”

x ∈ A “x is an element of A”

3. Set operations

3. Set operations

When we want to compare two sets together, we can look at their intersection and their union.

Intersection ∩
What do they have in common?

Union ∪
What are all the elements they have, combined?

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

3. Set operations

When doing an **intersection** or a **union** operation between two sets, the result is a third set.

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

3. Set operations

When doing an **intersection** or a **union** operation between two sets, the result is a third set.

A = { 1, 2, 3, 4 }

B = { 3, 5, 7 }

A ∩ B = What's in common?

A ∪ B = All elements together.

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

3. Set operations

When doing an **intersection** or a **union** operation between two sets, the result is a third set.

$A = \{1, 2, 3, 4\}$ $B = \{3, 5, 7\}$

$A \cap B = \{3\}$

$A \cup B = \{1, 2, 3, 4, 5, 7\}$

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

3. Set operations

If two sets have **nothing in common**, then the result is an empty set.

An empty set can be represented by {} OR by Ø *(NOT {Ø}!)*

Two sets with nothing in common are also called **Disjoint Sets**.

A = { 1, 2, 3, 4 }

C = { 5, 6, 7, 8 }

A ∩ B = Ø

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Disjoint Sets
Two sets with nothing in common.

Empty Set Ø or {}
A set with no elements.

3. Set operations

When we're working with a problem with multiple sets, we will use the **Universal Set U** to store all elements from all sets included in the problem.

A = { 1, 2, 3, 4 }

C = { 5, 6, 7, 8 }

B = { 10, 20 }

U = A ∪ B ∪ C

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Disjoint Sets
Two sets with nothing in common.

Empty Set Ø or {}
A set with no elements.

Universal Set
Contains all elements from all sets (possibly more).

3. Set operations

The **difference** of two sets A and B will be the set A, minus any elements from set B.

A = { 1, 2, 3, 4 }

B = { 1, 2 }

A – B = { 3, 4 }

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Disjoint Sets
Two sets with nothing in common.

Empty Set Ø or {}
A set with no elements.

Universal Set
Contains all elements from all sets (possibly more).

Difference A – B
Elements of A, except for elements from B.

3. Set operations

The **complement** of a set will be **all elements from the universe, without elements of that set.**

In other words, A’ = U – A.

**** The book writes a bar on top of a set for complement.***

$A = \{ 1, 2, 3, 4 \}$ *$U = \{ 1, 2, 3, 4, 5, 6 \}$*

A' or $\overline{A} = \{ 5, 6 \}$

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Disjoint Sets
Two sets with nothing in common.

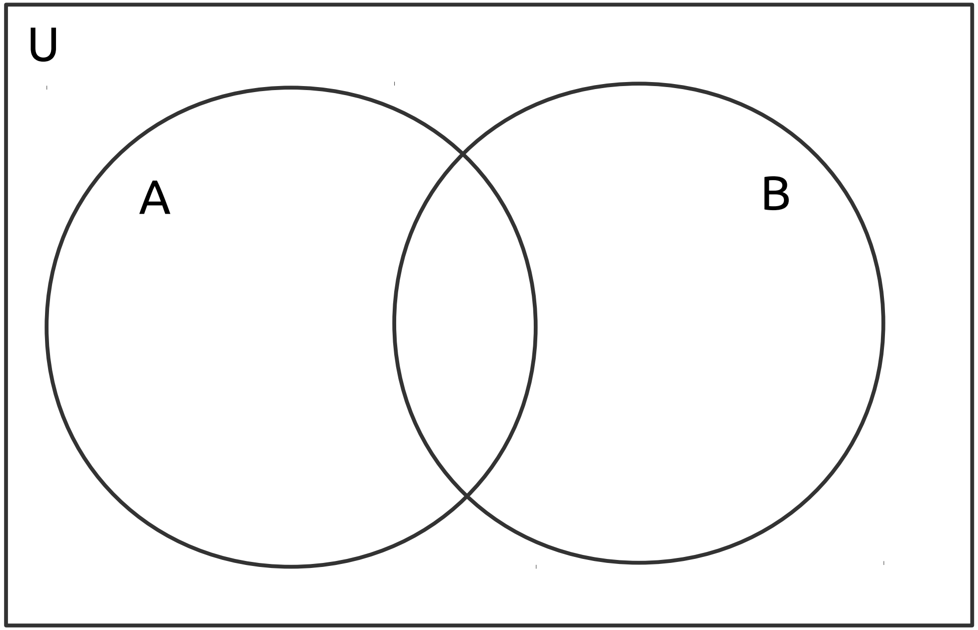
Empty Set Ø or {}
A set with no elements.

Universal Set
Contains all elements from all sets (possibly more).

4. Venn diagrams

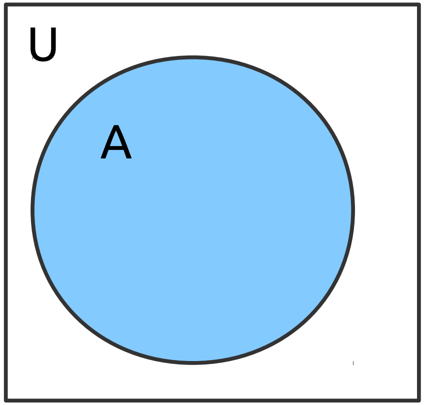
4. Venn diagrams

Venn Diagrams are a way we can visually show these set operations. A basic Venn diagram will have a box (The Universal Set), and a circle for each set in the problem.



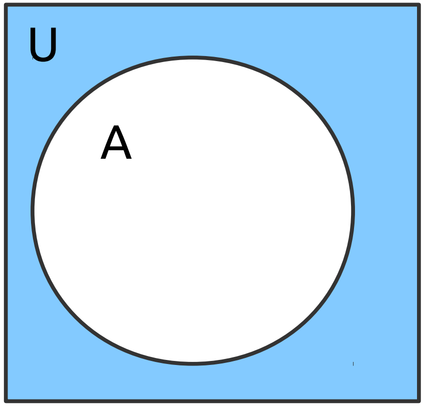
Notes

4. Venn diagrams



A

The Venn Diagram for *A* will just look like this – All of *A* is shaded in.



A'

A' will be all of the universe shaded in, EXCEPT for ANY of the *A* set.

Notes

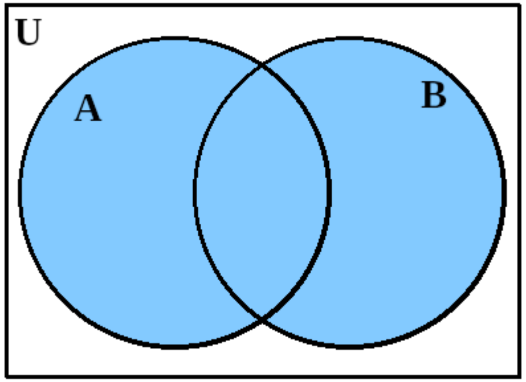
Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

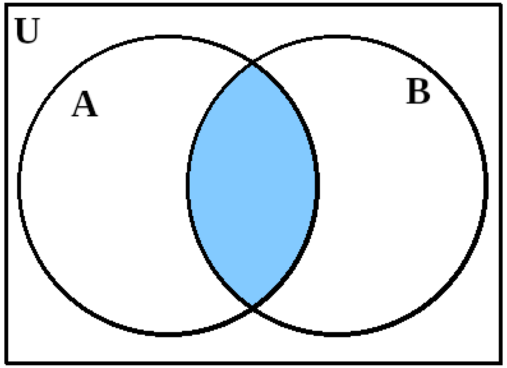
Universal Set
Contains all elements from all sets (possibly more).

4. Venn diagrams



A ∪ B ***A union B***

The Venn Diagram will have all of ***A*** and all of ***B*** shaded in, including their overlap.



A ∩ B ***A intersection B***

The Venn Diagram will have only what's in common between ***A*** and ***B*** – their overlap.

Notes

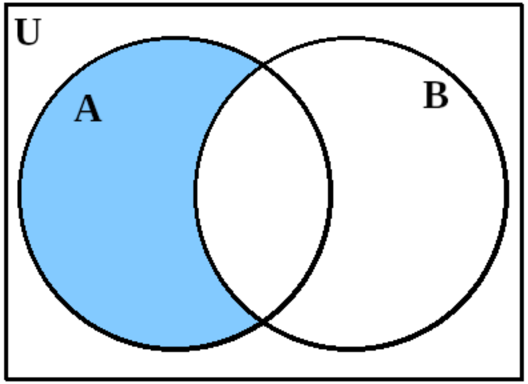
Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

4. Venn diagrams



A* - *B

The Venn Diagram will have all of ***A*** shaded, EXCEPT for anything in ***B***.

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

5. Complex Operations

Often we will be dealing with more than two sets, and working with the **intersection**, **union**, and **difference** operations, so let’s look at some examples.

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

1. $(A \cap B) \cup C$

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

1. (A ∩ B) ∪ C

A ∩ B = { 2, 3 }
(A ∩ B) ∪ C = { 2, 3, 6, 7 }

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

2. A ∩ C

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

2. A ∩ C

A ∩ C = {} **(empty set)**

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

3. A’ ∩ C

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

Given the following sets, find the results of the operations.

A = { 1, 2, 3, 4 } ***B = { 2, 3, 5 }*** ***C = { 6, 7 }***

U = { 1, 2, 3, 4, 5, 6, 7 }

3. A’ ∩ C

A’ = { 5, 6, 7 }
A’ ∩ C = { 6, 7 }

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

A ∩ B ∪ C - D ??

If you're inter-mixing the Union, Intersection, and/or Difference operations, you should use parentheses to make your intended order clear!

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

5. Complex Operations

(A ∩ (B ∪ C)) - D ??

If you're inter-mixing the Union, Intersection, and/or Difference operations, you should use parentheses to make your intended order clear!

Notes

Intersection - ∩
What is in common?

Union - ∪
All elements combined

Difference A – B
Elements of A, except for elements from B.

Universal Set
Contains all elements from all sets (possibly more).

6. DeMorgan’s Laws

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Just like how we have algebraic properties like…

$$a + b = b + a$$

$$a + (b + c) = (a + b) + c$$

… We also have laws for set operations. These are DeMorgan's laws…

Notes

asdfasdf

6. DeMorgan's Laws

Law	Property
Associative Laws	(A ∪ B) ∪ C = A ∪ (B ∪ C) (A ∩ B) ∩ C = A ∩ (B ∩ C)
Commutative Laws	A ∪ B = B ∪ A A ∩ B = B ∩ A
Distributive Laws	A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
Identity Laws	A ∪ Ø = A A ∩ U = A
Complement Laws	A ∪ A' = U A ∩ A' = Ø

Notes

6. DeMorgan’s Laws

Law	Property
Idempotent Laws	A ∪ A = A A ∩ A = A
Bound Laws	A ∪ U = U A ∩ Ø = Ø
Absorption Laws	A ∪ (A ∩ B) = A A ∩ (A ∪ B) = A
Involution Law	(A’)’ = A
0/1 Law	Ø’ = U U’ = Ø
DeMorgan’s Laws for Sets	(A ∪ B)’ = A’ ∩ B’ (A ∩ B)’ = A’ ∪ B’

Notes

Conclusion

This time we covered Set basics, operations, Venn diagrams, and DeMorgan’s Laws.

Next time we will continue working with sets as we look at Partitions, Power Sets, and Cartesian Products.