Set Operations

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About

We will be looking at three more topics relating to sets before we continue on to propositions – considering Partitions, generating Power Sets, and computing Cartesian Products.

Topics

1. Partitions
2. Power sets
3. Cartesian Products
4. Common pitfalls

1. Partitions

1. Partitions

The Partition of a set, usually denoted by **S**, is a <u>set of subsets</u> that, when combined together, form the original set.

Notes

A Partition of some set is <u>a set of subsets</u> where, when combined, form the original set.

1. Partitions

For a set ***A***, a partition of A is some set ***S* = {*S*$_1$, *S*$_2$, *S*$_3$, ...}** of subsets of ***A***, such that:

1) For all *i*, S_i != ∅

that is, each part is non-empty.

2) For all *i* and *j*, if S_i != S_j , then $S_i \cap S_j = \emptyset$

that is, different parts have nothing in common.

3) $S_1 \cup S_2 \cup S_3 \cup \ldots = A$

that is, every element in A is contained in some part.

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1. Partitions

Clarification:

- The partition is known as ***S***. Each element of ***S***, such as S_i , is known as a **part**.
- A **Part** is a set.
- No parts are empty sets, and all parts must have some elements that come from ***A***.
- An element of ***A*** cannot be repeated across multiple Parts.
- All elements of ***A*** must be represented in the entire partition ***S***.

Notes

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1. Partitions

Example:

Given a set $A = \{1, 2, 3, 4\}$, form a partition.

Notes

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1. Partitions

Example:
Given a set *A = {1, 2, 3, 4}*, form a partition.

Each of these are valid solutions:
- **{ {1}, {2}, {3}, {4} }**
- **{ {1, 2}, {3, 4} }**
- **{ {1, 2, 3}, {4} }**
- **{ {1, 2, 3, 4} }**

Note that each **part** is a set!

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2. Power Sets

2. Power Sets

The Power Set of ***A*** is defined as $\wp$***(A) = {S : S ⊆ A}***. In other words, the Power Set is the **set of all possible subsets** that you could build from ***A***, INCLUDING the empty set.

Notes

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A Power Set of some set is a set of all possible subsets that could be built from the original set.

2. Power Sets

Example:
Find the powerset $\wp$***({1})***

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2. Power Sets

Example:

Find the powerset $\wp(\{1\})$

$$\wp(\{1\}) = \{\varnothing, \{1\}\}$$

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Example:

Find the powerset $\wp(\{1\})$

$\wp(\{1\}) = \{\varnothing, \{1\}\}$

Find the powerset $\wp(\{1, 2\})$

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$\wp(\{1, 2\}) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$

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Find the powerset $\wp(\{1, 2, 3\})$

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Find the powerset $\wp(\{1, 2, 3\})$

$$\wp(\{1, 2, 3\}) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

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3. Cartesian Products

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We can compute the Cartesian Product of two sets, such as ***A*** and ***B***. The result will be a set of ordered pairs, such as ***(a, b)***, combining the elements of ***A*** and ***B*** together.

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The Cartesian Product of two sets is a set of ordered pairs, containing elements from the two sets.

3. Cartesian Products

Example:

For ***A* = {1, 2}** and ***B* = {4, 5, 6}**, find ***A* × *B***.

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3. Cartesian Products

Example:
For ***A* = {1, 2}** and ***B* = {4, 5, 6}**, find ***A* × *B***.

$A \times B = \{ (1, 4), (1, 5), (1, 6),$
$(2, 4), (2, 5), (2, 6) \}$

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4. Common Pitfalls

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Students frequently mix up Power Sets, Partitions, and Cartesian Products. Make sure that you understand the difference between each:

- **Partition of X**: A set of sets (parts), where each set contains some elements of the original set X. No sets can contain the same items, and no set can be empty. All elements of X must be represented in the partition.
- **Power Set of X**: A set of sets that gives you all possible subsets of the original set X, including the empty set.
- **Cartesian Product of X and Y**: A set of ordered pairs that combines all elements of X with all elements of Y .

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Conclusion

Chapter 1.1 covered a lot of information! You may wany to make sure to review all of 1.1 once studying for the first exam.

Next time, we will look at propositions and logical expressions: ***p and q***, ***p or q***, ***not p***, etc.